CDE/ECE Faculty Initiative Project Instructional Guide

Mathematics Domain of the *California Preschool Learning Foundations, Volume 1 (2008)*

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California Preschool Learning Foundations, Volume 1

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CDE/ECE Faculty Initiative Project
California Preschool Learning Foundations, Volume 1

Introduction to the Instructional Guide for the Mathematics Domain

This instructional guide is organized to support faculty in addressing the content and research base of the mathematics foundations. Accordingly, the instructional guide for these foundations is designed to support faculty as they deepen students' understanding of the structure, content, and research base of the foundations.

The guide is *not* intended to support faculty in helping students learn to *assess* children's learning and development related to the mathematics foundations. It is also *not* intended to support faculty in helping students learn how to design *curriculum* related to children's development of mathematics knowledge and skills. There are some activities that touch on these issues, but future instructional guides will address forthcoming California Department of Education preschool assessment and curriculum framework publications for these purposes.

The mathematics domain of the *California Preschool Learning Foundations, Volume 1* (PLF, V1*)* consists of five strands: (1) *Number Sense*; (2) *Algebra and Functions (Classification and Patterning)*; (3) *Measurement*; (4) *Geometry*; and (5) *Mathematical Reasoning*. The activities in this instructional guide allow faculty to address all the strands in an integrated approach or to focus on individual strands.

Consistent with the overall organization of the preschool instructional guides, the materials in this guide adhere to a framework of five core instructional components:

- Motivator and/or Connection to Experience
- Information Delivery
- In-Class Activity
- Out-of-Class Activity
- Assessment

These components are described fully in the overall introduction to this instructional guide.

Many activities include a suggestion for assessment of students' skills and/ or knowledge gained through the activity. Topics for additional study or research by students are included as *Deepening Understanding* activities at the end of the domain.

Most activities in the mathematics domain of the *Instructional Guide for the Preschool Learning Foundations, Volume 1* are to be done in class with suggestions included for out-of-class extensions. The intention is to thoroughly ground students in the structure,

content, and research base for these very important foundations and to build student confidence in their understanding of this domain.

It is suggested that faculty first use activities that directly explore the structure and content of the mathematics foundations before moving on to activities that use the foundations for further exploration of development in mathematics.

Organization Of Each Activity: Flexible Segments

Each activity is comprised of segments that allow faculty to make choices about implementation. The segments are italicized below as they are described.

Each activity begins with a section titled *Getting Ready*. This section can be found on the first page of every activity and provides an overview to help faculty decide if the activity fits into their purpose and goals for a class session. In this section there is a reference to the strands and a short description of the focus of the activity. A few tips for faculty preparation or reflection, labeled *Before you start*, are included before the activity. The activities themselves are divided into *Getting started, Keeping it going, Putting it together, Taking it further, Another way, Taking it out of class,* and *Assessment.* Not every activity includes all of these segments. Also note that assessments are included as segments within activities rather than as separate activities.

Throughout the Instructional Guide, you will sometimes see this symbol in the left margin below activity segments. This symbol indicates that there are PowerPoint slides that correspond to a particular part of the activity.

Slide 3

Knowledge and Skills

Each activity is accompanied by a description of the knowledge and skills students can gain from participating in that activity. The knowledge and skills can be found on the first page of each activity immediately following the *Getting Ready* section. The knowledge and skills associated with each activity might be useful in developing student learning outcomes, course objectives, or program objectives. Of course, the extent to which any student will attain the knowledge and skills will depend on both the breadth and depth with which faculty implement the activity.

These knowledge and skills have been aligned with early childhood professional standards from the National Association for the Education of Young Children (NAEYC) (see Appendix B) and the National Board of Professional Teaching Standards (NBPTS) (see Appendix C). The alignment with the core NAEYC professional standards uses the most recently revised edition (January 2009). In this revision, Standard 4 was divided into two separate standards, including one focused on teaching methods and the other on early childhood content (see Appendix B). Following each knowledge concept or skill, the notation in parentheses refers to the specific standard(s) from the *NAEYC Standards for Professional Preparation* with which it has been aligned. In addition, a matrix of the overlap between the NAEYC Professional Teaching Standards and the NBPTS has been included as a reference (see Appendix A). Please refer to Appendix A for the corresponding standards cited in the knowledge and skills for each activity.

Additional Thoughts

The activities in this guide are written to be adapted and, therefore, are not intended to be used as scripts. Each activity provides a framework within which faculty will need to plan and reflect on what will work best with their particular students. Several of the activities are straightforward examinations of the content of the strands, substrands, or foundations themselves. Others activities support deepening students' understanding of early mathematics as it is expressed in the foundations. This has the potential to bring some students to a deeper understanding of their own mathematics development, with sometimes challenging consequences. The extent to which any faculty develops these conversations will depend on the level of comfort and experience with these issues.

Much of early mathematics learning involves learning the language of mathematics, which means that language development strongly influences development of early mathematics concepts. Consequently, faculty might also benefit from being familiar with the foundations for English-language development and language and literacy, since they are closely related to the foundations for mathematics.

CDE has published a resource guide titled *Preschool English Learners: Principles ad Practices to Promote Language, Literacy, and Learning.* This guide provides foundational information regarding language and literacy development in all children, with special attention to English-language development in children for whom English is not their home language. Many faculty have found this publication to be helpful in supporting students who are learning about mathematics and the language of mathematics. The Faculty Initiative Project has produced an instructional guide for this publication, the *Instructional Guide for the Preschool English Learners: A Resource Guide*, which is available online at www.wested.org/facultyinitiative/pelguide.html.

Map of the Foundations

Mathematics

Domain

Strand

Algebra and Functions (Classification and Patterning)*

Age

Substrand

Foundations

At around 48 months of age	*At around 60 months of age*
1.0 Children begin to sort and classify objects in their everyday environment.	**1.0 Children expand their understanding of sorting and classifying objects in their everyday environment.**
1.1 Sort and classify objects by *one* attribute into two or more groups, with increasing accuracy.	**1.1** Sort and classify objects by *one or more* attributes, into two or more groups, with increasing accuracy (e.g., may sort first by one attribute and then by another attribute).†
Examples	**Examples**
• Selects some red cars for himself and some green cars for his friend, leaving the rest of the cars unsorted. • Chooses the blue plates from a variety of plates to set the table in the kitchen play area. • Sorts through laundry in the basket and takes out all the socks. • Places all the square tiles in one bucket and all the round tiles in another bucket. • Attempts to arrange blocks by size and communicates, "I put all the big blocks here and all the small ones there."	• Sorts the large blue beads into one container and the small red beads in another. • Puts black beans, red kidney beans, and pinto beans into separate bowls during a cooking activity. • Arranges blocks on the shelf according to shape. • Sorts a variety of animal photographs into two groups: those that fly and those that swim. • Sorts buttons first by size and then each subgroup by color into muffin tin cups.

Examples

* Throughout these mathematics foundations many examples describe the child manipulating objects. Children with motor impairments may need assistance from an adult or peer to manipulate objects in order to do things such as count, sort, compare, order, measure, create patterns, or solve problems. A child might also use adaptive materials (e.g., large manipulatives that are easy to grasp). Alternately, a child might demonstrate knowledge in these areas without directly manipulating objects. For example, a child might direct a peer or teacher to place several objects in order from smallest to largest. Children with visual impairments might be offered materials for counting, sorting, or problem solving that are easily distinguishable by touch. Their engagement is also facilitated by using containers, trays, and so forth that contain their materials and clearly define their work space.

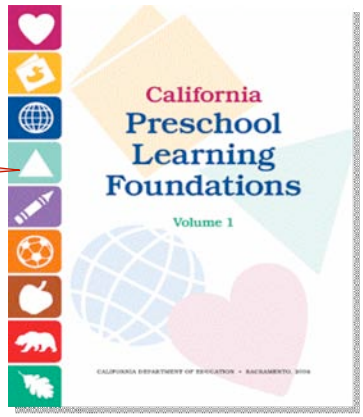
† Attributes include, but are not limited to, size, shape, or color.

Includes notes for children with disabilities

Mathematics Domain

California Preschool Learning Foundations Volume 1

Mathematics



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California Preschool Learning Foundations

1. Describe typical development, under conditions appropriate for healthy development, rather than aspirational expectations for children's behavior.
2. Assume learning for *all* children occurs in everyday environments through social interactions, relationships, activities, and play.
3. Are guidelines and teaching tools, *not* a list of items to be taught or used for assessment.

California Preschool Learning Foundations

- Are intended to be representative of and accessible to *all* learners, including children with disabilities and those learning English as a second language.
- Incorporate universal design for learning by encouraging *multiple and various* means of:
  - Representation
  - Engagement
  - Expression

Preschool Learning Foundations in Mathematics

- Identify behaviors in mathematics learning that are typical of children who will be ready to learn what is expected of them in kindergarten.
- Provide age-appropriate competencies expected or goals for *older* 3 year olds and *older* 4 year olds.
- Are intended to be guidelines and tools for instructional practice, not limits on the way teachers support children's learning at different levels.

School Readiness

- Focusing on the child's readiness for school in mathematics learning acknowledges that there must be appropriate social-emotional, cognitive, and language development, as well as appropriate motivation (p. 143, PLF, V1).

Map of the Foundations

Mathematics — Domain

Strand

Algebra and Functions (Classification and Patterning)*

Age

Substrand

Foundatio

At around 48 months of age	*At around 60 months of age*
1.0 Children begin to sort and classify objects in their everyday environment.	**1.0 Children expand their understanding of sorting and classifying objects in their everyday environment.**
1.1 Sort and classify objects by *one* attribute into two or more groups, with increasing accuracy.	1.1 Sort and classify objects by *one or more* attributes, into two or more groups, with increasing accuracy (e.g., may sort first by one attribute and then by another attribute).†
Examples	**Examples**
• Selects some red cars for himself and some green cars for his friend, leaving the rest of the cars unsorted. • Chooses the blue plates from a variety of plates to set the table in the kitchen play area. • Sorts through laundry in the basket and takes out all the socks. • Places all the square tiles in one bucket and all the round tiles in another bucket. • Attempts to arrange blocks by size and communicates, "I put all the big blocks here and all the small ones there."	• Sorts the large blue beads into one container and the small red beads in another. • Puts black beans, red kidney beans, and pinto beans into separate bowls during a cooking activity. • Arranges blocks on the shelf according to shape. • Sorts a variety of animal photographs into two groups: those that fly and those that swim. • Sorts buttons first by size and then each subgroup by color into muffin tin cups.

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† Attributes include, but are not limited to, size, shape, or color.

Includes notes for children with disabilities

Foundations in Mathematics: Five Developmental Strands

- Number Sense
- Algebra and Functions
- Measurement
- Geometry
- Mathematical Reasoning

Number Sense

- Important aspects of counting, number relationships, and operations (p. 145, PLF, V1).

Number Sense

At around 48 months of age	*At around 60 months of age*
1.0 Children begin to understand numbers and quantities in their everyday environment.	1.0 Children expand their understanding of numbers and quantities in their everyday environment.
2.0 Children begin to understand number relationships and operations in their everyday environment.	2.0 Children expand their understanding of number relationships and operations in their everyday environment.

Number Sense

- *Counting* is foundational for future understanding of mathematics.

- Building blocks for counting, include understanding:
  1. The sequence of number words.
  2. One-to-one correspondence.
  3. Cardinality.

Number Sense

- *One-to-one correspondence*
  - Understanding that one and only one number word is used for each object in the array of objects being counted.
- *Cardinality*
  - Knowing that the last number assigned to the last object counted gives the total number in the set.

Number Sense

- *Subitize*
  - The ability to quickly and accurately determine the quantity of objects in a small group (of up to five objects) without actually counting the objects.

Algebra and Functions (Classification and Patterning)

- Sorting and classifying objects; recognizing, extending, and creating patterns (p. 145, PLF, V1).

Algebra and Functions (Classification and Patterning)

Algebra and Functions

At around 48 months of age	*At around 60 months of age*
1.0.Children begin to sort and classify objects in their everyday environment.	1.0 Children expand their understanding of sorting and classifying objects in their everyday environment.
2.0 Children begin to recognize simple, repeating patterns.	2.0 Children expand their understanding of simple repeating patterns.

Algebra and Functions

- *Classification*
  - The sorting, grouping, or categorizing of objects according to an established criteria.
- *Attribute*
  - A property or characteristic of objects that can be used as a basis for grouping or sorting.

Measurement

- Comparing and ordering objects by length, weight, or capacity; precursors of measurement (p. 146, PLF, V1).

Measurement

At around 48 months of age	*At around 60 months of age*
1.0 Children begin to compare and order objects.	1.0 Children expand their understanding of comparing, ordering, and measuring objects.

Geometry

- Properties of objects (shape, size, position) and the relation of objects in space (p. 146, PLF, V1).

Geometry

At around 48 months of age	*At around 60 months of age*
1.0 Children begin to identify and use common shapes in their everyday environment.	1.0 Children identify and use a variety of shapes in their everyday environment.
2.0 Children begin to understand positions in space.	2.0 Children expand their understanding of positions in space.

Geometry

- *Shape knowledge*
  - The recognition, naming and understanding of shape characteristics and properties (Clements 2004a).
- *Space*
  - The recognition, naming and understanding of location, direction, distance and identification of objects (Clements, 1999).

Mathematical Reasoning

- Using mathematical thinking to solve problems in play and everyday activities (p. 147, PLF, V1).

Mathematical Reasoning

At around 48 months of age	*At around 60 months of age*
1.0 Children use mathematical thinking to solve problems that arise in their everyday environment.	1.0 Children expand the use of mathematical thinking to solve problems that arise in their everyday environment.

Mathematics:
Exploring Our Early Connections to the Mathematics Foundations

Strands:	***Number Sense***	***Algebra and Functions***	***Measurement***	***Geometry***	***Mathematical Reasoning***

GETTING READY

Instructional Component(s): Motivator and/or Connection to Experience; In-Class Activity

Strands: This activity can be used to develop familiarity with and deepen understanding of all math strands or of individual strands.

Focus: Students explore their own early experiences with mathematics and how those experiences might affect their work in supporting young children's development in mathematics.

AFTER PARTICIPATING IN THIS ACTIVITY

Students will demonstrate ***knowledge*** and ***skills*** that are consistent with an increased understanding and application of the concepts addressed in this activity.

Students will demonstrate an understanding of:
- How their own experiences learning mathematics may affect the ways in which they help children engage with math (*Standards 1, 4, & 6*)*

Students will be able to:
- Reflect on their own experiences learning math (*Standards 4 & 6*)*
- Consider how their own experiences affect how they support young children's development in mathematics (*Standards 1, 4, & 6*)*

*See Appendix A

Mathematics:
Exploring Our Early Connections to the Mathematics Foundations

Before you start

Research indicates that understanding mathematics, beyond the intuitive and informal experiences that many children have, is critical to future academic success. However, many students and practitioners may not fully engage young children with mathematics experiences because they are uncertain about their own knowledge and skills relating to mathematics. Consequently, many students find it hard to engage with the concepts and possibilities of helping young children's development relating to mathematical concepts.

Because of this, it is often helpful to begin exploring the mathematics foundations by delving into our own relationship to math.

Getting it started

Slide 2

Organize students into pairs or groups of three. Keeping the groups small will increase the likelihood that ideas and feelings will surface. Be sensitive to the depth at which these stories and memories may elicit an emotional response and do not require that all students retell their memories.

Ask students to respond to these questions:

- When you reflect on your own experiences learning mathematics, what specific memories come to mind? Consider particular classes, instructors, activities, or homework assignments.
- Do you remember a time when you really *enjoyed* mathematics? Again, consider particular classes, instructors, activities, or homework that may have been particularly interesting or rewarding.
- Do you remember a time when you first thought math was hard and you could not do it? Again, consider particular classes, instructors, activities, or homework that may have posed a challenge for you.

Keeping it going

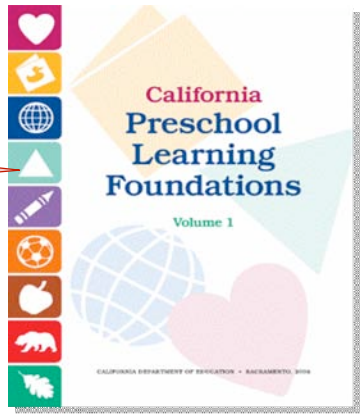
Ask students to think of two or three words or phrases that describe their feelings about learning math. Ask them to write these individually on large Post-it® notes or half sheets that can be taped to a board or chart paper.

Putting it together	When all the words are visible, ask the class to review the words that they generated They can be read aloud by one student or shared by several. Then, ask students to group these words together with other words that are similar or the same. Sometimes it helps to ask, "Do you see any that are similar or express a similar feeling or idea?"
Taking it further	Ask them to come up with names for each group of words. Sometimes the words or phrases fall neatly into two categories—*positive* and *challenges*—though there are many other categories they could fall into, such as *feeling challenged*, *important events*, or *enjoyment*.
Targeting individual strands	You might ask students to reflect on their experience with this question: • What do our own experiences tell us about working with young children to enhance their development in mathematics concepts and skills? You might point out that, by organizing into groups with similar characteristics and naming the groups, they have just engaged in a classification activity that is related to the algebra and functions strand of the math foundations.

Mathematics Domain

California Preschool Learning Foundations
Volume 1

Mathematics



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Exploring Our Connections to the Math Foundations

- When you reflect on your own experiences learning mathematics, what specific memories come to mind? (classes, instructors, activities)
- Do you remember a time when you really enjoyed mathematics?
- Do you remember a time when you first thought math was *hard* and you could not do it?

Mathematics:
Exploring the Mathematics Foundations in Our Daily Lives

Strands:	***Number Sense***	***Algebra and Functions***	***Measurement***	***Geometry***	***Mathematical Reasoning***

GETTING READY

Instructional Component(s): Motivator and/or Connection to Experience; In-Class Activity

Strands: This activity can be used to develop familiarity with and deepen understanding of all math strands or of individual strands.

Focus: Students explore how they themselves and the children with whom they work demonstrate skills and concepts in the math foundations in their daily routines.

AFTER PARTICIPATING IN THIS ACTIVITY

Students will demonstrate ***knowledge*** and ***skills*** that are consistent with an increased understanding and application of the concepts addressed in this activity.

Students will demonstrate an understanding of*:*
- How math is used in daily life (*Standards 4 & 5*)*
- How children experience math throughout everyday interactions and routines, as well as formal instruction (*Standards 1 & 5*)*
- The foundations in mathematics, including strands, substrands, foundations, and examples of foundations (introductory understanding) (*Standards 1, 5, &* 6)*

Students will be able to:
- Consider the prevalence and importance of mathematics in daily life (*Standards 4 & 5*)*
- Identify activities during a typical day that involve math (*Standard 5*)*
- Identify ways children experience math in their daily lives (*Standards 1 & 5*)*
- Identify and describe examples of how adults and children use number sense (classification and patterning, measurement, geometry, and mathematical reasoning) in daily life (*Standards 1 & 5*)*
- Connect their own and children's daily experiences with the foundations in mathematics (*Standards 1, 4, & 5*)*
- Reflect on their own experiences with math concepts and skills (*Standards 4, 5, & 6*

*See Appendix A

Mathematics:
Exploring the Mathematics Foundations in Our Daily Lives

Before you start

Sometimes it is helpful to reinforce the importance of mathematics to us and to young children by pointing out that we use math informally and almost intuitively throughout our daily lives. In addition to our intentional teaching, children accumulate a great deal of understanding of mathematical concepts in their everyday interactions and routines.

Following is a two-part activity that will focus students on the importance of math in our lives, ways that math is used in daily life, and ways that children experience the math concepts in the foundations during their everyday lives.

Getting it started

In pairs or small groups, ask students to consider what math they use in their everyday lives. Ask students to take a piece of paper, divide it into two columns, and label one column "What I Did" and the other "Foundation Referenced." You could also provide them with the worksheet at the end of this activity.

Ask students to document the activities from their day that used or required math knowledge and then decide which strand of the math foundations is relevant to each activity by writing it in the corresponding column to the right.

To help initiate this thinking process, ask the whole group to consider what they did during the first half hour after waking up or the half hour immediately before they came to class. Ask for some student volunteers to share one thing that occurred in this half hour period so other students will have an example to follow.

Here are some possible routines that might help students connect their lives to mathematics:

- Did they check on the time?
- Did they run a shower?
- Did they fix something to eat or drink?

If students need an alternative way to think about this, you might want to prompt them with specific routines, such as cooking, shopping, driving, doing laundry, staying on schedule, listening to music, dancing, or playing games. You also might ask them to consider the activities of their paid work if they have a job during the day other than caring for children.

Keeping it going

Then ask students to work in pairs and continue documenting the math in their day. If you want to focus their attention on specific strands, refer them to strands, substrands, and/or foundations and ask them to think of ways in which they were demonstrated, either actively or as background knowledge. Refer to the following strands specifically:

- Counting or Number Operations
- Classification or Patterning
- Measurement
- Geometry
- Mathematical Reasoning

Putting it together

Slides 2-3

After about 15 minutes, ask students to look at their lists and think about the following questions:

- What was the strand most frequently listed? Least frequently listed? Why do you think that is the case?
- What does this tell us about math concepts and skills in our own daily lives?
- What does this tell us about the importance of mathematics learning in ECE?
- What does this tell us about some of the ways that children gain experience with the math concepts in the math foundations?

Another way

Slide 4

Ask students to consider what math young children have participated in or experienced before they come to school in the morning and/or between the time they go home and their bedtime.

Again, ask students to take a piece of paper and divide it into two columns. Label one “What Young Children Do” and the other “Foundation Referenced.” Or you can have students use the worksheet provided. Ask students to document a child’s activities that require mathematical knowledge and then decide which strand of the math foundations is relevant to each activity by writing it in the corresponding column to the right.

Targeting individual strands

Targeting any one of the strands and its substrands and/or foundations can be done in either of the following ways:

- Assign each pair of students to one strand and ask them to think of how, during the day, they used skills and/or knowledge related to that strand

Have the whole class work on one strand only. This might be appropriate in a curriculum or observation class that is addressing that strand. For example, if students are being asked in a curriculum class to develop materials or activities related to supporting children's engagement with counting, they might use this activity as a link to their own experience with that strand.

What I Did	Foundation Referenced

What Young Children Do	Foundation Referenced

Mathematics Domain

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Exploring the Math Foundations in Our Daily Lives

- What was the strand most/least frequently listed? Why?
- What does this tell us about math concepts and skills in our own daily lives?

Exploring the Math Foundations in Our Daily Lives

- What does this tell us about the importance of mathematics learning in ECE?
- What does this tell us about some of the ways that children gain experience with the math concepts in the math foundations?

Exploring the Math Foundations in Our Daily Lives

What Young Children Do	Foundations Referenced

Mathematics:
Finding Mathematics All Around Us

Strands:	***Number Sense***	***Algebra and Functions***	***Measurement***	***Geometry***	***Mathematical Reasoning***

GETTING READY

Instructional Component(s): In-Class Activity; Assessment

Strands: This activity can be used to develop familiarity with and deepen understanding of all math strands or of individual strands.

Focus: Students recognize that the concepts and skills described in the mathematics foundations occur in everyday situations and in natural environments.

AFTER PARTICIPATING IN THIS ACTIVITY

Students will demonstrate ***knowledge*** and ***skills*** that are consistent with an increased understanding and application of the concepts addressed in this activity.

Students will demonstrate an understanding of*:*

- How mathematical concepts can be explored using objects found in our everyday settings (*Standard 5*)*
- Possible ways to use objects to engage children who are English learners with math concepts (*Standards 1 & 4*)*
- The foundations in mathematics, including strands, substrands, foundations, and examples of foundations (introductory understanding) (*Standards 1, 5, & 6*)*

Students will be able to:

- Recognize that the concepts and skills in the math foundations occur in everyday situations (*Standard 4*)*
- Consider how to engage children with mathematics through the use of everyday objects (*Standards 1 & 4*)*
- Support children who are English learners to develop skills and understanding in mathematics (*Standards 1, 4, & 5*)*

*See Appendix A

Mathematics:
Finding Mathematics All Around Us

Before you start

It is helpful to demonstrate that mathematical concepts can be richly discussed and described with objects that we frequently encounter.

Ask students to bring to class a collection of natural objects (e.g., nuts, leaves, rocks, feathers, or sticks) for use in this activity.

Getting it started

Assign students in pairs or small groups to a strand or more than one strand. Have students spread their natural objects out on a table or floor area, and let them know that their explorations will demonstrate the mathematics foundations. Remind students that foundations are about children's development rather than curriculum.

Ask that they take 10 to 15 minutes to decide ways they can demonstrate to others in the class how they can address the concepts in the math foundations while exploring these materials.

You could add other props that would allow students to consider single variables, such as shape, or two variables simultaneously, such as size and color or shape and color.

Keeping it going

Slides 2-6

There are many possibilities for this activity, but here are some examples of questions that could be provided to students to facilitate the work:

Number Sense
- What could they count?
- Where could they find groups or separate into groups?
- What operations could they use?

Measurement
- What attributes of the objects could be measured?
- What could these objects be compared to or measured against?

Algebra and Functions
- How can they be grouped (classified)?
- How could patterns be seen or made?

Geometry
- What shapes can be recognized?
- What shapes can be made?

Mathematical Reasoning
- What can they build with these objects?
- How can they make equal groups?

Putting it together

Ask each group to demonstrate their work. Their ideas can be recorded and later put together as a resource.

Taking it further

Ask students to consider how they would engage students who are English learners in an activity like this. How could they connect children who are English learners to the rich vocabulary that is used here?

Another way

Slide 7

Sometimes it is interesting to bring in different sets of objects, such as buttons, and ask how these could be used to demonstrate these ideas, as well.

Then ask this question:

- How might younger preschool children experience this differently from older preschool students?

Assessment

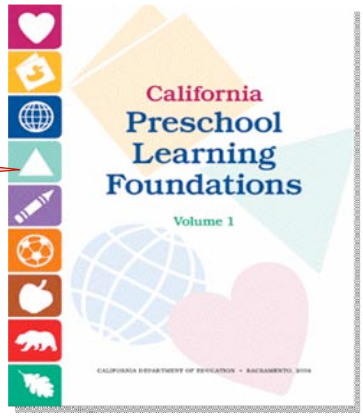
If collections of objects found in nature are used for this activity, other types of objects, such as the buttons, could be used for assessment. The same questions could be used, and students could record their responses. This would require that they not only remember what happened with the natural objects but also think about additional or different possibilities.

Students could also complete the activity out of class and submit their written results.

Mathematics Domain

California Preschool Learning Foundations
Volume 1

Mathematics



Published by the
California Department of Education (2008)

Mathematics All Around Us

Number Sense

- What could they count?
- Where could they find groups or separate into groups?
- What operations could they use?

Mathematics All Around Us

Measurement

- What attributes of the objects could be measured?
- What could these objects be compared to or measured against?

Mathematics All Around Us

Algebra and Functions

- How can they be grouped/classified?
- How could patterns be seen or made?

Mathematics All Around Us

Geometry

- What shapes can be recognized?
- What shapes can be made?

Mathematics All Around Us

Mathematical Reasoning

- What can they build with these objects?
- How can they make equal groups?

Mathematics All Around Us

- How might younger preschool children engage mathematically with natural objects differently than older children?

Mathematics:
Piecing Together the Mathematics Domain Puzzle

Strands:	***Number Sense***	***Algebra and Functions***	***Measurement***	***Geometry***	***Mathematical Reasoning***

GETTING READY

Instructional Component(s): In-Class Activity; Assessment

Strands: This activity can be used to develop familiarity with and deepen understanding of all math strands or of individual strands.

Focus: Students become familiar with the organization and content of the mathematics foundations.

AFTER PARTICIPATING IN THIS ACTIVITY

Students will demonstrate ***knowledge*** and ***skills*** that are consistent with an increased understanding and application of the concepts addressed in this activity.

Students will demonstrate an understanding of:

- The foundations in mathematics, including strands, substrands, foundations, and examples of foundations (introductory understanding) (*Standards 1, 5, & 6*)*

Students will be able to:

- Demonstrate and discuss the different knowledge and skills contained in the foundations in mathematics, including identifying examples of the foundations at different ages in preschool development (*Standards 1, 5, & 6*)*

*See Appendix A

Mathematics:
Piecing Together the Mathematics Domain Puzzle

Before you start

This activity is designed to help students become familiar with the content and organization of the mathematics domain of the foundations. This exercise is useful as a beginning exploration of these foundations or at the end of study as an assessment. It also could be used at any point in the process of familiarization as a self-assessment.

Create puzzles for your students by pulling the foundations apart. One way to make your own puzzle is to make a large card for each strand: *Number Sense, Algebra and Functions, Measurement, Geometry,* and *Mathematical Reasoning*. Then make a paper strip for each of the substrands under each strand and then strips for the foundations under the substrands. A template has been created and is available at the end of this activity. It is helpful to put each set into an envelope that can easily be distributed to groups or individual students.

Getting it started	Distribute sets of the foundations that have been assembled in envelopes. You will need as many sets as there will be groups, pairs, or, if they will do this individually, individual students. Ask your students to reassemble the sets. For each strand, ask students to place appropriate substrands under or next to it. Next ask them to place appropriate foundations under or next to the substrands.
Keeping it going	If you choose to do this with small groups, five groups would give each group a chance to focus on one strand. Instead of having each group limited to one strand, you could also have each group rotate through the five strands.
Putting it together  Slides 2-3	If your exploration of the foundations takes place over several weeks, you could also have students explore one strand each week. Ask students to reflect on their experience. • Were some strands easier to put together than others? Why do you think this is so? • What was most challenging? Most fun?

- Can you think of a way to do this with family members in an evening group meeting?
- Can you think of examples they have seen of one or two of the foundations in each strand?
- Which substrand do you want to know more about? How could you find out more about that strand?

Taking it further

Students can prioritize the substrands according to which strands they believe are most important for teachers to know about and be able to work on.

Where do they see the most need for teachers to support development in the children they work with?

Assessment

Putting the puzzle together could be used as an assessment activity. If this is done as an assessment, students' work can be checked and recorded as appropriate to your particular needs and procedures.

For example, if students are doing this early in their work on the math domain of the foundations, you might want to allow students the support of being able to check their work with the *California Preschool Learning Foundations, Volume 1 (PLF, V1)*. Another form of support is to allow them to check the work of a peer or the work of other groups.

This also could be used before beginning work with the math foundations, using the publication as a guide, and then again after working with them as an assessment, without using the publication as a guide. Upon completion, students should check all of their work with the publication.

Number Sense	
At around 48 months of age	*At around 60 months of age*
Children begin to understand numbers and quantities in their everyday environment.	Children expand their understanding of numbers and quantities in their everyday environment.
Recite numbers in order to ten with increasing accuracy.	Recite numbers in order to twenty with increasing accuracy.
Begin to recognize and name a few written numerals.	Recognize and know the name of some written numerals
Identify, without counting, the number of objects in a collection of up to three objects (i.e., subitize).	Identify, without counting, the number of objects in a collection of up to four objects (i.e., subitize).
Count up to five objects, using one-to-one correspondence (one object for each number word) with increasing accuracy.	Count up to ten objects, using one-to-one correspondence (one object for each number word) with increasing accuracy.
Use the number name of the last object counted to answer the question, "How many . . . ?"	Understand, when counting, that the number name of the last object counted represents the total number of objects in the group (i.e., cardinality).
Children begin to understand number relationships and operations in their everyday environment.	Children expand their understanding of number relationships and operations in their everyday environment.
Compare visually (with or without counting) two groups of objects that are obviously equal or nonequal and communicate, "more" or "same."	Compare, by counting or matching, two groups of up to five objects and communicate, "more," "same as," or "fewer" (or "less").

Understand that adding to (or taking away) one or more objects from a group will increase (or decrease) the number of objects in the group.	Understand that adding one or taking away one changes the number in a small group of objects by exactly one.
Understand that putting two groups of objects together will make a bigger group.	Understand that putting two groups of objects together will make a bigger group and that a group of objects can be taken apart into smaller groups.
Solve simple addition and subtraction problems nonverbally (and often verbally) with a very small number of objects (sums up to 4 or 5).	Solve simple addition and subtraction problems with a small number of objects (sums up to 10), usually by counting.

Algebra and Functions *(Classification and Patterning)*	
At around 48 months of age	*At around 60 months of age*
Children begin to sort and classify objects in their everyday environment.	Children expand their understanding of sorting and classifying objects in their everyday environment.
Sort and classify objects by one attribute into two or more groups, with increasing accuracy.	Sort and classify objects by one or more attributes, into two or more groups, with increasing accuracy (e.g., may sort first by one attribute and then by another attribute).
Children begin to recognize simple, repeating patterns.	Children expand their knowledge of simple, repeating patterns.
Begin to identify or recognize a simple repeating pattern.	Recognize and duplicate simple repeating patterns.
Attempt to create a simple repeating pattern or participate in making one.	Begin to extend and create simple repeating patterns.

Measurement	
At around 48 months of age	*At around 60 months of age*
Children begin to compare and order objects.	Children expand their understanding of comparing, ordering, and measuring objects.
Demonstrate awareness that objects can be compared by length, weight, or capacity, by noting gross differences, using words such as *bigger, longer, heavier,* or *taller,* or by placing objects side by side to compare length.	Compare two objects by length, weight, or capacity directly (e.g., putting objects side by side) or indirectly (e.g., using a third object).
Order three objects by size.	Order four or more objects by size.
	Measure length using multiple duplicates of the same-size concrete units laid end to end.

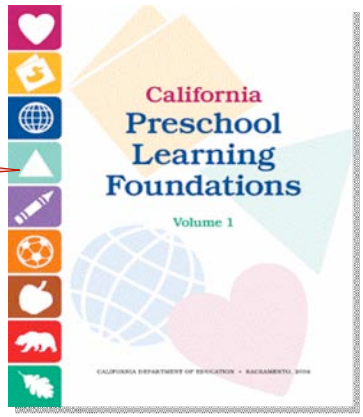
Geometry	
At around 48 months of age	*At around 60 months of age*
Children begin to identify and use common shapes in their everyday environment.	Children identify and use a variety of shapes in their everyday environment.
Identify simple two-dimensional shapes, such as a circle and square.	Identify, describe, and construct a variety of different shapes, including variations of a circle, triangle, rectangle, square, and other shapes.
Use individual shapes to represent different elements of a picture or design.	Combine different shapes to create a picture or design.
Children begin to understand positions in space.	Children expand their understanding of positions in space.
Identify positions of objects and people in space, such as in/on/ under, up/down, and inside/outside.	Identify positions of objects and people in space, including in/on/ under, up/down, inside/outside, beside/between, and in front/behind.

Mathematical Reasoning	
At around 48 months of age	*At around 60 months of age*
Children use mathematical thinking to solve problems that arise in their everyday environment.	Children expand the use of mathematical thinking to solve problems that arise in their everyday environment.
Begin to apply simple mathematical strategies to solve problems in their environment.	Identify and apply a variety of mathematical strategies to solve problems in their environment.

Mathematics Domain

California Preschool Learning Foundations Volume 1

Mathematics



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California Department of Education (2008)

Piecing Together the Mathematics Domain Puzzle

- Were some strands easier to put together than others? Why?
- What was most challenging? Most fun?

Piecing Together the Mathematics Domain Puzzle

- Can you think of a way to do this with family members?
- Can you think of examples of one or two of the foundations in each strand?
- Which substrand do you want to know more about? How could you find out more about that strand?

Mathematics:
Understanding the Research and Evidence Base for the Mathematics Domain

Strands:	***Number Sense***	***Algebra and Functions***	***Measurement***	***Geometry***	***Mathematical Reasoning***

GETTING READY

Instructional Component(s): Information Delivery; Out-of-Class Activity; Assessment

Strands: This activity can be used to develop familiarity with and deepen understanding of all math strands or of individual strands.

Focus: Students explore the research base of the foundations.

AFTER PARTICIPATING IN THIS ACTIVITY

Students will demonstrate ***knowledge*** and ***skills*** that are consistent with an increased understanding and application of the concepts addressed in this activity.

Students will demonstrate an understanding of*:*
- The research underlying the PLF, V1 (introductory understanding) (*Standards 1 & 6*)*
- How bibliographic notes can be a useful introduction to core research ideas (*Standard 6*)*

Students will be able to:
- Use bibliographic notes to become introduced to research underlying the PLF, V1 (*Standard 6*)*
- Consider the implications of research findings on practice (*Standards 1-6*)*

*See Appendix A

Mathematics:
Understanding the Research and Evidence Base for the Mathematics Domain

Before you start

The *California Preschool Learning Foundations, Volume 1* (PLF, V1), includes a section of bibliographical notes for each domain. These bibliographic notes provide summaries of the research and evidence that underlie the foundations. Understanding that the foundations are based upon research is a very important concept. Many of the research papers in the bibliographic notes are foundational to the field of early childhood development and education. Additionally, many of the research articles give students an overall appreciation for cross-cultural studies of math education in other cultures or countries.

Getting it started

Assign students individually, in pairs, or small groups to a strand. Ask students to find and read the bibliographic notes in the PLF, V1 relating to the particular strand that they were assigned. Ask them to find three to five key phrases or ideas from the notes and be ready to report them to the whole class.

It will be helpful to record the phrases or sentences as they are reported to the whole group. This will be most helpful if done by strand and if the questions in the following section are asked for each strand.

Putting it together

Slide 2

You might want to ask students to consider these questions for each strand:

- What was new to you?
- Is there one phrase or idea that seems most important?
- How would one's cultural community or language affect how children demonstrate these capacities?
- What are the implications for how you engage young children in early experiences with mathematics?

Another way

This activity could be done all in one class period or spread out over several classes, using one strand for each class. It also could be implemented by posting the sentences or phrases from each group online. This would give students time to consider the responses to the questions in the preceding section.

Taking it out of class

Students could be assigned to find one article in the bibliographic notes and respond to the questions listed previously relating to that article. You might want to provide a selected list of these articles, as there is a range of breadth and depth and all might not be appropriate for a particular group of students.

Reviewing the *Deepening Understanding* section in this instructional guide will provide ways to involve students more deeply with relevant research.

Additionally, a review of the literature relating to any one strand would be suitable for advanced students.

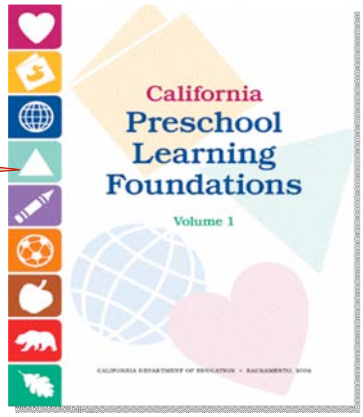
Assessment

This activity could be done as an individual written assignment completed out of class. In that case, consider having each student review all the strands and perhaps find three key ideas in each strand. Students' responses could still be used for class discussion as described in this activity.

Mathematics Domain

California Preschool Learning Foundations
Volume 1

Mathematics



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Understanding the Research and Evidence Base

Consider for each strand:

- What was new to you?
- Is there one phrase or idea that seem most important?
- How would one's cultural community or language affect how children demonstrate these capacities?
- What are the implications for how you engage young children in early experiences with mathematics?

Mathematics:
Linking Children's Home Experiences with the Mathematics Foundations

Strands:	***Number Sense***	***Algebra and Functions***	***Measurement***	***Geometry***	***Mathematical Reasoning***

GETTING READY

Instructional Component(s): In-Class Activity; Out-of-Class Activity; Assessment

Strands: This activity can be used to develop familiarity with and deepen understanding of all strands or of individual strands.

Focus: Students explore the vocabulary of early mathematics in the preschool setting and at home.

AFTER PARTICIPATING IN THIS ACTIVITY

Students will demonstrate ***knowledge*** and ***skills*** that are consistent with an increased understanding and application of the concepts addressed in this activity.

Students will demonstrate an understanding of:

- How math is used in daily life (*Standards 4 & 5*)*
- How children experience math throughout everyday interactions and routines, as well as formal instruction (*Standards 1 & 5*)*
- The foundations in mathematics, including strands, substrands, foundations, and examples of foundations (introductory understanding) (*Standards 1, 5, & 6*)*
- How different contexts affect children's experiences with mathematics (*Standards 1 & 5*)*

Students will be able to:

- Consider the prevalence and importance of mathematics in daily life (*Standards 4 & 5*)*
- Identify activities during a typical day that involve math (*Standard 5*)*
- Identify ways children experience math in their daily lives (Standards *1 & 5*)*
- Identify and describe examples of how adults and children demonstrate number sense (classification and patterning, measurement, geometry, and mathematical reasoning) in daily life (*Standards 1 & 5*)*
- Connect their own and children's daily experiences with the foundations in mathematics (*Standards 1 & 5*)*
- Reflect on their own experiences with math concepts and skills (*Standards 4, 5, & 6*)*
- Communicate with family members about how they use math in their daily lives (*Standard 2*)*
- Identify common opportunities for experiences in math between the classroom setting and those reported by families from their daily lives (Applies to "Taking it Further") (*Standards 1, 2, 4, & 5*)*

*See Appendix A

Mathematics:
Linking Children's Home Experiences with the Mathematics Foundations

Before you start

It is important for students and/or practitioners to forge connections with the ways in which math is experienced at home and to connect families to the ways in which math is experienced in the preschool. Several possible ways to do this are suggested here.

Getting it started

Slides 2-3

Ask students to interview members of their own family or families of young children with whom they work about their use of math at home.

Prepare some interview questions together in class. For example, students might ask the following questions:

- What are some ways that measurement is used in your everyday life (cooking, buying food)?
- What are some ways that counting and numbers are used in your everyday life? (Where do you add or subtract things? Where do you use one-to-one correspondence?)
- Where are patterns and classifying (matching, sorting, grouping) used in your everyday life?
- Where is geometry (shapes, positions in space) used in your everyday life (driving, putting dishes away)?

Keeping it going

Slides 4-5

Some prompts might be these questions:

- What size and weight words are related to being at the grocery store?
- How do we use comparison words at the grocery store?
- What size, shape, or sorting and/or pattern words relate to doing laundry?
- What measurement words relate to cooking?
- Where and what do we count during the day?

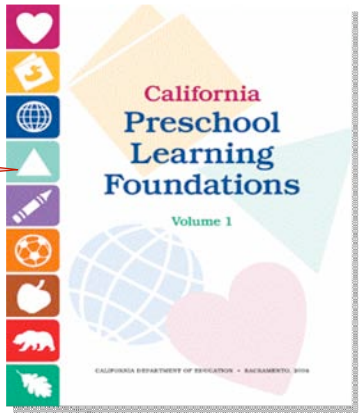
Taking it further

Consider asking questions about work-related uses of mathematics. For example, you might ask, "How do family members use math in a particular profession or work environment?" Some occupations to consider are construction worker, retail clerk, computer engineer, medical technician, truck driver, farm worker, car mechanic, farmer, nurse, and teacher.

	If you ask the same questions listed previously about work-related use of math, you might get longer and more complex lists. These would help highlight the importance of mathematics in our lives but might be harder to relate to early childhood uses of everyday mathematics.
Putting it together	Ask students to bring the examples back to class, where you can link their examples to specific foundations. One way to do this would be to have a chart paper for each strand posted or on tables and ask students to write their examples on these papers. These lists could also be composed online. Ask students to consider how the vocabulary words that preschool children are learning relate to the examples provided from the families.
Taking it out of class	If you have access to a preschool classroom, ask students to take these examples from home life and walk through the classroom to see where these same math skills are used in the everyday environment of the early childhood classroom. Students could be given a checklist of the examples that were generated in their interviews with parents, with space below the checklist or in a second column to list examples in the classroom. Again provide an opportunity for students to share their results and link to specific foundations.
Assessment	Students could be asked to take the information from these activities and apply it to working with families. Ask students to use the information to plan a family evening related to the math foundations. Have students focus on involving families in understanding the links between home and school mathematics in the early years. Intentional curriculum features could also be demonstrated here but might require a previous course in curriculum development or that this activity is done as part of a curriculum course.

Mathematics Domain

California Preschool Learning Foundations
Volume 1



Mathematics

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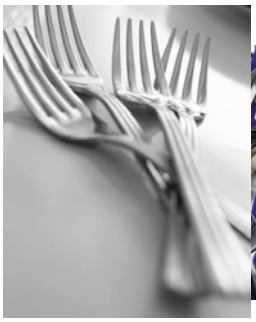
California Department of Education (2008)

Linking Children's Home Experiences with the Mathematics Foundations

- What are some ways that measurement is used in your everyday life?
- What are some ways that counting and numbers are used in your everyday life?

Linking Children's Home Experiences with the Mathematics Foundations

- Where are patterns and classifying used in your everyday life?
- Where is geometry used in your everyday life?



Linking Children's Home Experiences with the Mathematics Foundations

- What size and weight words are related to being at the grocery store?
- How do we use comparison words at the grocery store?

Linking Children's Home Experiences with the Mathematics Foundations

- What size, shape, or sorting and/or pattern words relate to doing laundry?
- What measurement words relate to cooking?
- Where and what do we count during the day?

Mathematics:
Learning Mathematics in the Context of Our Cultural Communities

Strands:	***Number Sense***	***Algebra and Functions***	***Measurement***	***Geometry***	***Mathematical Reasoning***

GETTING READY

Instructional Component(s): Motivator and/or Connection to Experience; In-Class Activity

Strands: This activity can be used to develop familiarity with and deepen understanding of all mathematics strands or of individual strands.

Focus: Students explore ways in which cultural communities affect how young children learn mathematical skills and concepts by exploring their own childhood experiences that relate to the mathematics strands.

AFTER PARTICIPATING IN THIS ACTIVITY

Students will demonstrate ***knowledge*** and ***skills*** that are consistent with an increased understanding and application of the concepts addressed in this activity.

Students will demonstrate an understanding of:
- The ways context and culture, including experiences with our communities, language, and family, influence how children (and we as adults) learn math (*Standard 1*)*
- How children gain knowledge and understanding by building on prior experiences with their family and community (*Standard 1*)*
- Why familiar contexts and materials help children to apply mathematical concepts and use strategies (*Standards 1 & 5*)*

Students will be able to:
- Reflect upon how math was used in their family and community while growing up (*Standards 4, 5, & 6*)*
- Consider the impact of context and culture on development (*Standard 1*)*
- Think about and discuss typical mathematical experiences of preschool children (*Standards 1 & 5*)*

*See Appendix A

Mathematics:
Learning Mathematics in the Context of Our Cultural Communities

Before you start

An important point for practitioners to understand about math is that, as children and as adults, we learn math concepts and skills in the context of our cultural communities. We can begin to uncover some of this by asking students how they remember math being used in their own family as they were growing up. This provides opportunities to explore cultural manifestations of mathematics.

Getting it started

Ask students to consider family contexts to begin to understand how math was used in their families.

- Do they remember seeing their parents or other significant adults using math for household or occupational purposes?
- Do they remember using math with siblings? Did they play games that used math or use math as they did chores with siblings?
- How was math talked about and used in their home?
- Are there sayings, songs, or rhymes that they learned as children that involved counting, adding, or subtracting?
- Do they remember seeing or hearing a family member estimate when they couldn't be accurate or precise?
- Do they remember times when they made things equal or compared amounts of things?

This can be explored for the mathematics foundations overall or for each of the five strands. If you choose to implement this activity for each strand individually, the following examples suggest some questions that could be used.

Targeting individual strands

Slides 2-6

Number Sense

- How did you learn to count?
- Were there things in your family that were counted regularly?
- Where or when did you add and subtract amounts?

Algebra and Functions

- Did you sort or match things with your parents? Laundry? Dishes?
- Do you remember patterns in rugs, tiles, or linoleum floors?

Measurement

- Do you remember some things you saw measured or that you measured yourself?
- Do you remember watching or hearing family members compare size, weights, or shapes of things?
- What kinds of tools did you see your family use to measure?
- Did you organize things by size?

Geometry

- Did you fold laundry?
- Do you remember shapes in rugs, tiles or windows?
- Did you sort things into different drawers and cupboards of different shapes?

Putting it together

Slide 7

If you choose to review these as a whole group, it would be interesting to find similarities and differences in the experiences of the students. If you have a diverse group of students, you might ask students to consider how differences in their math experiences are related to differences within and across languages, families, and geographic and economic communities.

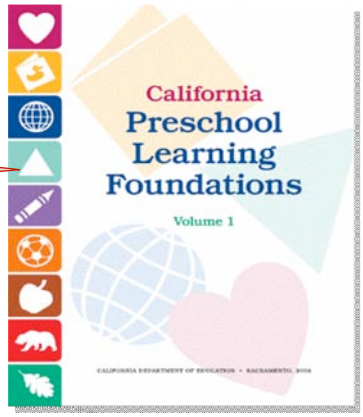
Ask students to consider these questions:

- What do these differences in our own experiences tell us about the mathematical experiences of the children who come to us as learners?
- In what ways might language and culture affect our early mathematics experiences?

Mathematics Domain

California Preschool Learning Foundations
Volume 1

Mathematics



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Learning Math in the Context of Our Cultural Communities

Number Sense

- How did you learn to count?
- Were there things in your family that were counted regularly?
- Where or when did you add and subtract amounts of things?

Learning Math in the Context of Our Cultural Communities

How was math used in your family?

Learning Math in the Context of Our Cultural Communities

Algebra and Functions

- Did you sort or match things with your parents?
- Do you remember patterns in your house?

Learning Math in the Context of Our Cultural Communities

Measurement

- Do you remember some things you saw measured or that you measured?
- Do you remember watching or hearing family members compare size, weights, or shapes of things?
- What kinds of tools did you see your family use to measure?
- Did you organize things by size?

Learning Math in the Context of Our Cultural Communities

Geometry

- Did you fold laundry?
- Do you remember shapes in and around your house? (rugs, tiles, windows)
- Did you sort things into different drawers and cupboards of different shapes?

Learning Math in the Context of Our Cultural Communities

- What does this tell us about the mathematical experiences of the children who come to us as learners?
- In what ways might language and culture affect our early mathematical experiences?

Mathematics:
Exploring Examples of the Mathematics Foundations

Strands:	***Number Sense***	***Algebra and Functions***	***Measurement***	***Geometry***	***Mathematical Reasoning***

GETTING READY

Instructional Component(s): Information Delivery; In-Class Activity; Out-of-Class Activity

Strands: This activity can be used to develop familiarity with and deepen understanding of all math strands or of individual strands.

Focus: Students observe and record examples of young children demonstrating their development of skills and concepts found in the math foundations.

AFTER PARTICIPATING IN THIS ACTIVITY

Students will demonstrate ***knowledge*** and ***skills*** that are consistent with an increased understanding and application of the concepts addressed in this activity.

Students will demonstrate an understanding of*:*

- The foundations in mathematics, including strands, substrands, foundations, and examples of foundations (introductory understanding) (*Standards 1, 5, & 6*)*
- Preschool children's skills in mathematics as demonstrated in an early childhood setting (*Standards 1 & 5*)*
- Preschool children's developmental progression of mathematics understanding and skills (*Standards 1 & 5*)*

Students will be able to:

- Identify and describe examples of preschool children demonstrating number sense, classification and patterning, measurement, geometry, and mathematical reasoning in an early childhood setting (*Standards 1 & 5*)*
- Begin to differentiate between the skills of children who are "around 48 months" and those who are "around 60 months of age" (*Standards 1 & 5*)*

*See Appendix A

Mathematics:
Exploring Examples of the Mathematics Foundations

Before you start

Become familiar with the examples in the foundations and have some examples of your own ready to start the conversation. Consider using video examples, if available.

Be aware that your approach to this activity can strengthen students' understanding that the examples in the publication are not the only ways in which children will demonstrate these skills and concepts.

Getting it started

Slide 2

Spend some time examining the examples provided for each of the math foundations. Help students understand that the examples that are provided are representative examples and are not exhaustive. It is also important to help students understand that the examples are not criteria for the foundations.

The purpose of this exercise is not to assess children's development but to explore the foundations and see where and how children engage with the skills and concepts. We expect to see children engaging with these concepts and skills in a variety of ways and with varied materials. As you review the examples in the math foundations, ask students for examples that they have seen in young children.

As an observation activity, ask students to build ongoing example banks for the math foundations. Ask students to observe children in a preschool classroom. Provide students with a list of the math strands, substrands, and foundations. Ask them to look for examples of children demonstrating the use of these foundations in their play and/or conversation or other engagement with the concepts of the foundations. Be sure to remind them that they are looking for specific statements or actions by the children.

This could be done with on-site observations of 30 minutes in each of two areas of an early childhood setting. For example, one student could choose art and water play and do an observation in each area for 30 minutes. You would want to make sure that you have a variety of areas covered. You also might assign each student to a strand or one strand per area. The strand of *Mathematical Reasoning* might be addressed separately or in conjunction with any of the other strands. It could actually be helpful to have more than one student doing each of the strands.

Keeping it going

Remind the students that, when they look for examples in classrooms in which they are working or with children they know, they are not looking for skill levels but just for examples of the foundations.

They might be reluctant to judge whether what they see is appropriate for "at around 48 months" or "at around 60 months of age." Suggest that they write down what they see, bring it back to class, and be ready to talk about it.

Ask students to bring their lists of examples to class. Each student can write the examples out on strips of paper or in a list on chart paper for the class to see. Alternatively, the students could submit their examples online prior to class, and these could be printed out and distributed to all the students. Organize the examples is such a way that they are displayed in relation to the strand they exemplify.

Putting it together

Slide 3-4

Give students time to walk around and read each other's examples.

Ask questions such as these examples:

- Is there anything you particularly noticed about all the examples?
- Were there some areas or activities where it was difficult to see children engaging with math?
- Were there some where it was easy?
- Were skills and behaviors in some strands easier to see than others? Which ones? Why?
- What does this tell you about child development?
- What are the implications of this for including math in the ECE curriculum?

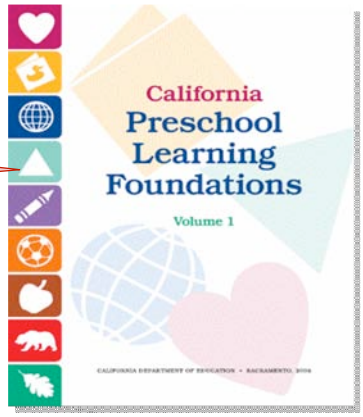
Since the focus here is not curriculum, keep responses general, such as the need to provide materials, the need to be intentional about planning and also looking for teachable moments.

Be sure that the examples are collected and made available as a resource to students.

Mathematics Domain

California Preschool Learning Foundations
Volume 1

Mathematics



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Example Bank

What examples of the mathematics foundations have you observed in preschool children?

Consider….

- Interactions
- Verbalizations
- Behaviors

FIP IG Math Activity 8

Example Bank

- Is there anything you notice?
- Were there areas or activities where it was difficult to see children engaging with mathematics?
- Were there some where it was easy?

Example Bank

- Were skills and behaviors in some strands easier to see than others? Why?
- What does this tell you about child development?
- What are the implications for including mathematics in the ECE curriculum and environment?

Mathematics:
Understanding How Children Demonstrate the Mathematics Foundations in Early Care and Education Settings

Strands:	***Number Sense***	***Algebra and Functions***	***Measurement***	***Geometry***	***Mathematical Reasoning***

GETTING READY

Instructional Component(s): In-Class Activity; Out-of-Class Activity

Strands: This activity can be used to develop familiarity with and deepen understanding of all mathematics strands or of individual strands.

Focus: Students engage with activities that are typical of those that young children experience in early childhood settings. By interacting with materials and engaging in routines, they directly experience the everyday mathematics in early care and education settings.

AFTER PARTICIPATING IN THIS ACTIVITY

- The foundations in mathematics, including strands, substrands, foundations, and examples of foundations (introductory understanding) (*Standards 1, 5, & 6*)*
- Where and how children experience mathematics in their early childhood education programs (*Standards 4 & 5*)*
- The developmental progression of knowledge and skills in mathematics *(applies to "Taking it out of class")* (*Standards 1 & 5*)*

Students will be able to:

- Connect specific foundations and related strands to typical activities in an early childhood education setting (*Standards 1, 4, & 5*)*
- Identify where and how children engage in and demonstrate skills and behaviors described in the foundations in mathematics during a typical day in an early childhood setting (*Standards 1, 4, & 5*)*
- Recognize children experiencing mathematics in an early childhood setting (*applies to "Taking it out of class"*) (*Standards 1 & 3*)*
- Identify knowledge and skills in mathematics that are characteristic of younger and older preschool children (*applies to "Taking it out of class"*) (*Standards 1 & 5*)*

*See Appendix A

Mathematics:
Understanding How Children Demonstrate the Mathematics Foundations in Early Care and Education Settings

Before you start

To help students see where children are regularly engaging with the content of the math foundations in early care and education programs, provide the opportunity to experience typical activities from an early childhood setting.

Set up the room with materials for typical activities and routines from an early childhood group setting. For example, place materials such as these on different tables: play dough, manipulatives, art materials, dress-up items, blocks, books, snack materials, and water play objects. If you have space, also set up an obstacle course.

You will need to have envelopes containing all the substrands. If you have done the activity titled "Piecing Together the Mathematics Domain Puzzle," you will have these already prepared. You can also use the worksheet at the end of this activity. You can use foundations themselves, but because of the number it might be better to work at the substrand level.

Getting it started

Place an envelope containing strips of all the math substrands at each table. These strands can be found in a worksheet at the end of this activity. Remind students not to open the envelopes immediately. Have students spend 10-15 minutes working with the materials at that table. After they have spent 10-15 minutes working with the materials, ask them to take out the strips and find the math foundations that children would experience while engaging with those materials.

Keeping it going

Depending on the time available and the goal for this activity, you might consider having students rotate through the stations to help them identify a variety of foundations. This might also allow for discussion between groups who visited the same table but who may have identified different foundations at that table.

If groups are rotating from table to table, ask them to list the foundations identified at each station on a sheet of paper and place all the strips back into the envelope so that the next group can use them.

Putting it together

Once students have completed this activity, reconvene the group and ask them to list the foundations for each table so that all can review them. You might do this by distributing chart paper to each table and asking students to tape their list of foundations to each chart paper or write down what is on their lists. Provide an opportunity for students to rotate again and see what foundations each group associated with each table.

Targeting individual strands

Slides 2-4

Help students to connect the specific foundations and related strands to their experiences at the tables by asking the following question.

At which table or with which materials would children be gaining experience with each strand:

- Number Sense
- Algebra and Functions
- Measurement
- Geometry
- Mathematical Reasoning

Follow this with a conversation about the experience.

- What stands out to you as you look at the lists that have been generated?
- What does this tell us about how and where children experience math in the early childhood classroom?
- If you rotated to a second table, were you more aware of what math might be experienced there? Why?

Taking it further

As an important extension, ask students to consider times and activities during the day not represented by the activities they just did when children might engage with each of the math strands.

Some topics are particularly rich, such as the following activities:

- Cleanup
- Outdoor play
- Meals or snack
- Water or sand table*
- Obstacle course*

They could do this with an assigned time or activity, or they could brainstorm generally and their ideas could be recorded and organized as they are generated in the group.

Again it might be helpful to have cut-out strips, lists or tables with the foundations available so that it is easier for students to correlate activities with substrands found in the foundations.

* Math opportunities at the water or sand table and obstacle course would be best experienced if they could be set up for exploration, but if not, they are still important to explore in the brainstorming activity.

Taking it out of class

This activity could also be done as an out-of-class observation activity. Ask students to observe some of the activity centers in a preschool program for 15-20 minutes. Anecdotal recording of what children do and say would be a rich way to document this observation. Students could then use their notes as evidence of children demonstrating the development as referenced in the foundations.

If they can do this with both preschool children of differing ages, it will help them see the developmental progression that children go through during preschool years as described in the mathematics foundations.

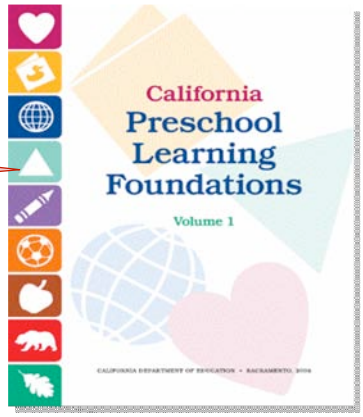
Substrands from the Mathematics Domain

Children begin to understand numbers and quantities in their everyday environment.	**Children expand their understanding of numbers and quantities in their everyday environment.**
Children begin to understand number relationships and operations in their everyday environment.	**Children expand their understanding of number relationships and operations in their everyday environment.**
Children begin to sort and classify objects in their everyday environment.	**Children expand their understanding of sorting and classifying objects in their everyday environment.**
Children begin to recognize simple, repeating patterns.	**Children expand their understanding of simple, repeating patterns.**
Children begin to compare and order objects.	**Children expand their understanding of comparing, ordering, and measuring objects.**
Children begin to identify and use common shapes in their everyday environment.	**Children identify and use a variety of shapes in their everyday environment.**
Children begin to understand positions in space.	**Children expand their understanding of positions in space.**
Children use mathematical thinking to solve problems that arise in their everyday environment.	**Children expand the use of mathematical thinking to solve problems that arise in their everyday environment.**

Mathematics Domain

California Preschool Learning Foundations
Volume 1

Mathematics



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How Children Demonstrate the Mathematics Foundations

At which table, or with which materials, can children gain experience with:

- Number sense
- Algebra and functions
- Measurement
- Geometry
- Mathematical reasoning

How Children Demonstrate the Mathematics Foundations

- What stands out to you as you look at the lists that have been generated?

How Children Demonstrate the Mathematics Foundations

- What does this tell us about how and where children experience mathematics in the early care and education setting?
- If you rotated to a second table, were you more aware of what mathematics might be experienced there? Why?

Mathematics:
Using Weekly Songs, Finger Plays, or Dances to Explore the Mathematics Foundations

Strands:	***Number Sense***	***Algebra and Functions***	***Measurement***	***Geometry***	***Mathematical Reasoning***

GETTING READY

Instructional Component(s): In-Class Activity

Strands: This activity can be used to develop familiarity with and deepen understanding of all math strands or of individual strands.

Focus: Students experience a variety of ways to engage young children with the concepts and skills of the math foundations through song, rhyme, and dance.

AFTER PARTICIPATING IN THIS ACTIVITY

Students will demonstrate ***knowledge*** and ***skills*** that are consistent with an increased understanding and application of the concepts addressed in this activity.

Students will demonstrate an understanding of*:*

- The foundations in mathematics, including strands, substrands, foundations, and examples of foundations (introductory understanding) (*Standards 1, 5, & 6*)*
- Ways in which children experience mathematics in their early childhood education programs (*Standards 4 & 5*)*
- How gaining understanding and skills in math relates to the development of language and literacy and social-emotional development (*Standard 1*)*

Students will be able to:

- Identify where and how during a typical day in an early childhood setting children demonstrate knowledge and skills described in the foundations in mathematics (*Standards 1, 4, & 5*)*
- Identify and/or develop songs, finger plays, chants, or dances that engage children with the concepts of the math foundations (*Standards 1, 4, &* 5)*

*See Appendix A

Mathematics:
Using Weekly Songs, Finger Plays, or Dances to Explore the Mathematics Foundations

Before you start

As an ongoing activity, consider having a song or dance and finger play of the week. In addition to considering which math foundations are represented in these activities, these activities present a good opportunity to consider the relation of math concepts and skills to the development of language and literacy and to social-emotional development.

Consider asking for songs in home languages as well as in English. Be sure to collect and print the songs and finger plays and document the dances so that all students can have them as resources for their teaching.

Getting it started

Ask students to bring to class a song, finger play, chant, or dance that will engage young children with the concepts of the math foundations. Students could be assigned to bring these to class individually or in pairs and assigned to a particular class session during which they will demonstrate the activity.

Remind students to find songs, finger plays, or dances that involve the five strands of the math foundations. Ask them to find songs, finger plays, or dances that involve counting, measuring, shapes, patterns, grouping, or solving a problem. They could be assigned to particular strands or bring in something that relates to any strand. Over time, it would be helpful to be sure that all strands have been included in what the students are bringing to class.

Putting it together

Slide 2

The students' examples can be used to demonstrate the prevalence of math concepts in music, dance, and finger plays. Try to encourage dance, music, and finger plays in several languages and from a variety of cultural communities if they are represented in your students or in the children with whom they will likely be working.

Students sometimes find it challenging to recognize math concepts in dance or music, so ask students where they could see patterning, counting, repetition, use of spatial relations, use of size and shape, and mathematical reasoning in the music and dances.

You may find it helpful to ask students to reflect on what they have heard or seen and describe the specific foundations that are experienced in a specific piece of music, rhyme or movement. You may also want to ask students to explain or consider why a song, finger play or dance might be more appropriate for younger or older preschool children.

Taking it further

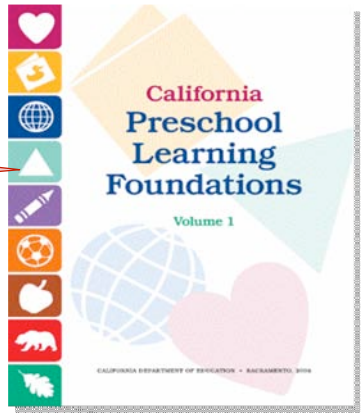
If any students are involved in groups that do traditional music or dances of their home culture, ask if they could demonstrate the music or some of those dances.

If there is no one in the class who can do this, ask students in pairs to invent a dance or movement that incorporates some of these math components, such as counting, patterns, and repetitions. Emphasize that students should design this dance to be used with preschool children.

Mathematics Domain

California Preschool Learning Foundations Volume 1

Mathematics



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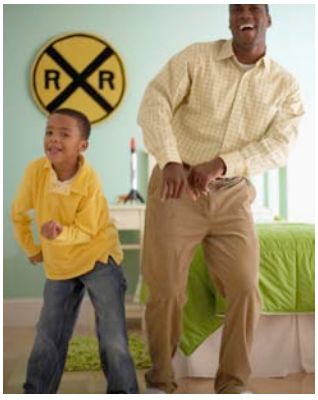
Using Songs, Finger Plays, and Dances to Explore the Math Foundations

When do you notice…

- Patterning
- Counting
- Repetition
- Use of spatial relations
- Use of size and shape
- Mathematical reasoning
- Other math concepts



Mathematics:
Working With Blocks to Explore the Mathematics Foundations

Strands:	***Number Sense***	***Algebra and Functions***	***Measurement***	***Geometry***	***Mathematical Reasoning***

GETTING READY

Instructional Component(s): In-Class Activity

Strands: This activity can be used to develop familiarity with and deepen understanding of all mathematics strands or of individual strands.

Focus: Students engage with the rich possibilities of using blocks to develop math vocabulary, concepts, and skills in young children.

AFTER PARTICIPATING IN THIS ACTIVITY

Students will demonstrate ***knowledge*** and ***skills*** that are consistent with an increased understanding and application of the concepts addressed in this activity.

Students will demonstrate an understanding of*:*

- The vocabulary and concepts embedded in the foundations in mathematics, including strands, substrands, foundations, and examples of foundations (introductory understanding) (*Standards 1 & 5*)*
- Where and how children experience mathematics in their early childhood education programs (*Standards 4 & 5*)*
- How blocks can be used to engage young children with math skills and concepts (*Standards 1, 4, & 5*)*
- The relationship of language to developing knowledge and skills in mathematics (*Standard 1*)*
- The foundations for social-emotional development (initial overview) (*Standards 1 & 5*)*
- How aspects of social-emotional development can be related to developing skills and behaviors in mathematics (*applies to "Taking it further")* (*Standard 1*)*
- The interrelatedness of domains of development, including examples from all the foundations in the PLF, V1 (social-emotional development, language and literacy, English-language development, and mathematics) (*applies to "Taking it further")* (*Standard 1*)*

Students will be able to:

- Identify how during a typical activity (blocks) in an early childhood setting children engage in and demonstrate skills and behaviors described in the foundations in mathematics (*Standards 1, 4, & 5*)*
- Use the vocabulary and concepts from the foundations in mathematics (*Standards 1 & 5*)*
- Consider how to use activities in the block area to support children who are English learners build vocabulary relating to mathematics (*Standards 1, 4, & 5*)*
- Connect the foundations in social-emotional development to a group block building activity (*applies to "Taking it further")* (*Standards 1 & 5*)*

*See Appendix A

Mathematics:
Working With Blocks to Explore the Mathematics Foundations

Before you start

Blocks have been seen for many years as a particularly fruitful activity for engaging young children with math skills and concepts. This activity provides students with an opportunity to engage with blocks in the way that young children do, and to discover the language, concepts, and skills that are relevant to the math foundations as children build with blocks. It can also be helpful in demonstrating that language is critical to early engagement with mathematics.

Getting it started

Organize students into small groups of three or four and give each group some blocks. If you are able to give each group a different amount you might get more variety in the results. Give students about ten minutes to build whatever they want with the blocks you have given them.

Keeping it going

If convenient, you also might want to use different sized blocks of differing materials, if convenient. If a preschool classroom is nearby, try this activity after the children are gone, or consider borrowing soft foam blocks, uniform cubes, or other types of building blocks.

Students could also be given patterns to try to replicate, such as building patterns or photographs of other block structures. blocks in wood, plastic, rubber, or other materials [Note – did not understand why these types of blocks are in this sentence – recommend deleting)

Putting it together

Slides 2-3

When students have finished, ask them the following questions:

What vocabulary related to the math foundations do you remember using as you built your structure?

- Number Sense: Did you do any counting and/or comparison of quantity?
- Geometry: Did you name and/or compare shapes?
- Measurement: Did you discuss or compare size, volume or weight?
- Algebra and Functions (Classification and Patterning): Did you put patterns together? Did you talk about symmetry? Did you sort by any characteristics of the blocks?
- Mathematical Reasoning: Did you analyze what would work and what would not?

As they recall the vocabulary words relating to the foundations, record the words for all to see. Keep this list for further work below.

Taking it further

Next ask students to think about what they would do in relation to these vocabulary words if they have children who are English learners. How could they use the block area to build English vocabulary relating to mathematics in all students? For example, it would be important for adults working with the children to know some of these words in the home language of the children.

In addition, some of these words might be posted near the block area in both home language and English so that they could be used and paired as children are working with blocks.

Another way

This activity also can be accomplished by assigning a scribe to each small group so that words are recorded as they are used. While this method might collect more vocabulary words, it also makes students more self-conscious of their conversation and can produce more contrived results

You could also ask students to take about 10-15 minutes in their group and quickly review the social-emotional development foundations. As they do so, ask them to think about which ones apply to their group's building activity.

Ask students to reflect on the implications of this for introducing math concepts to young children.

Targeting individual strands

Slide 4

If students have completed all three phases of this activity, they have touched on all the foundation domains in the *California Preschool Learning Foundations, Volume 1* (language and literacy, social-emotional development, English-language development, and mathematics).

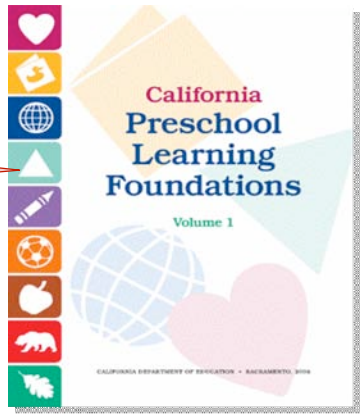
As you look at the math strands, ask these questions:

- Which of the strands from other domains support the development of skills within each math strand?
- Which is co-developing?
- Which is strengthened?

Mathematics Domain

California Preschool Learning Foundations
Volume 1

Mathematics



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Working with Blocks

Number sense

Did you do any counting and/or comparison of quantity?

Geometry

Did you name and/or compare shapes?

Measurement

Did you discuss or compare size, volume, or weight?

Working with Blocks

Algebra and Functions (Classification and Patterning)

Did you put patterns together? Did you talk about symmetry? Did you sort by any characteristics of the blocks?

Mathematical Reasoning

Did you analyze what would work and what would not?

Working with Blocks

Consider the relatedness of the Foundations….

	Language and Literacy	Social-Emotional	English-Language Development
Number Sense			
Algebra and Functions			
Measurement			
Geometry			
Mathematical Reasoning			

Mathematics:
Exploring the Vocabulary of Early Mathematics

Strands:	***Number Sense***	***Algebra and Functions***	***Measurement***	***Geometry***	***Mathematical Reasoning***

GETTING READY

Instructional Component(s): In-Class Activity; Out-of-Class Activity

Strands: This activity can be used to develop familiarity with and deepen understanding of all math strands or of individual strands.

Focus: Students explore the vocabulary of early mathematics in the preschool setting and at home, and its connection to early language learning.

AFTER PARTICIPATING IN THIS ACTIVITY

Students will demonstrate ***knowledge*** and ***skills*** that are consistent with an increased understanding and application of the concepts addressed in this activity.

Students will demonstrate an understanding of:

- How math is used in daily life (*Standards 4 & 5*)*
- Common vocabulary associated with mathematics (*Standard 5*)*
- How children experience math throughout everyday interactions and routines, as well as formal instruction (*Standards 1 & 5*)*
- The foundations in mathematics, including strands, substrands, foundations, and examples of foundations (introductory understanding) (*Standards 1, 5, & 6*)*
- How different contexts affect children's encounters with math vocabulary and concepts (*applies to "Taking it further")* (*Standards 1 & 5*)*

Students will be able to:

- Identify common vocabulary associated with mathematics (*Standard 5*)*
- Consider the usage of math vocabulary throughout typical daily activities and routines in an early childhood setting (*Standard 5*)*
- Identify and describe examples of how adults and children use number sense (classification and patterning, measurement, geometry, and mathematical reasoning) in daily life (*Standards 1 & 5*)*
- Communicate with family members about how they use math in their daily lives (applies to "*Taking it further*") (*Standard 2*)

*See Appendix A

Mathematics:
Exploring the Vocabulary of Early Mathematics

Before you start

It may be helpful to have this activity follow "Learning Mathematics in the Context of Our Cultural Communities". While that activity engages students in thinking about their own use of mathematics, this activity explores the use of mathematics and math vocabulary by children and their families.

Getting it started

Ask students to develop lists of vocabulary words related to the foundations in each strand. One way to do this is to organize students into groups of three or four. Assign each group to a strand in the foundations. Ask them to write a list as fast as possible of all the words they can think of that capture the math concepts of that strand.

Some words that students might suggest are faster, slower, longer, shorter, bigger, smaller, big, medium, small, smallest, largest, tall, taller, tallest, heavy, heaviest, light, lighter, lightest, wide, widest, long, longer, longest, minute, second, hour, day, week, circle, triangle, square, rectangle, parallelogram, oval, 3-D, sphere, cube, pyramid, and cylinder.

Keeping it going

Post the lists and rotate the strands at least once to new groups. Ask them to add whatever they can to the list that is posted.

Putting it together

Slides 2-3

Then ask the following questions:

- How do the words for each strand relate to the activities, materials, and routines that children typically do in a preschool classroom?

If you have done the activity entitled "Understanding How Children Demonstrate the Mathematics Foundations in Early Care and Education Settings," you can refer back to those particular preschool activities to help students respond to this question.

- What does this tell us about the importance of language development to early math development?
- How might this affect children who are English learners?
- What does this suggest about our role as teachers and/or important adults in the lives of young children?

Taking it out of class

Slides 4-6

Have students interview members of their own family or families of young children with whom they work.

Ask families about some ways in which they use the following math concepts in their everyday life:

- Measurement (cooking, buying food)
- Number Sense/Counting (Where do they add or subtract things? Where do they use one-to-one correspondence?)
- Algebra/Classification and Patterning (matching, sorting, grouping)
- Geometry (shapes, positions in space)

Some prompts might be these questions:

- What size and weight words are related to being at the grocery store?
- How do we use comparison words at the grocery store?
- What size, shape, or sorting and/or pattern words relate to doing laundry?
- What measurement words relate to cooking?
- Where and what do we count during the day?

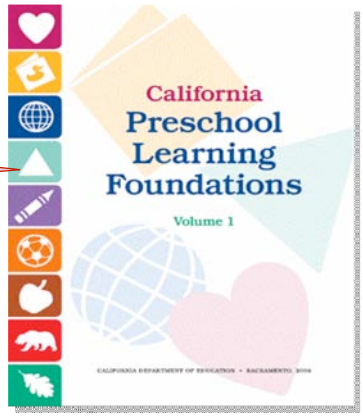
Ask students to consider how the vocabulary words that preschool children are learning relate to the examples provided from the families.

Taking it further

Also see the activity titled “Linking Children’s Home Experiences with the Mathematics Foundations” as a way to extend or deepen the understanding of how home, cultural, and language experiences influence the development of mathematics vocabulary.

Mathematics Domain

California Preschool Learning Foundations
Volume 1



Mathematics

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California Department of Education (2008)

Exploring the Vocabulary of Early Mathematics

- How do the words in each strand relate to the activities, materials, and routines that children typically do at preschool?

Exploring the Vocabulary of Early Mathematics

- What is the importance of language development to early mathematics development?
- How might this affect children who are English learners?
- What are the implications for our role as teachers and/or important adults in the lives of young children?

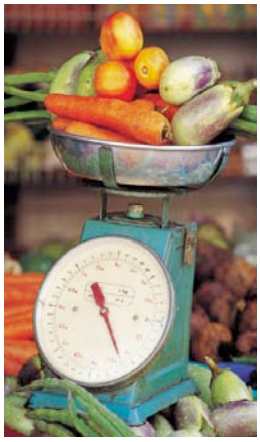
Exploring the Vocabulary of Early Mathematics

Ask families about some ways in their everyday lives when they use:

- Measurement
- Counting
- Algebra
- Geometry

Exploring the Vocabulary of Early Mathematics

Some prompts for families might be:



- What size and weight words are related to being at the grocery store?
- How do we use comparison words at the grocery store?

Exploring the Vocabulary of Early Mathematics

Some prompts for families might be:

- What size, shape, or sorting and/or pattern words relate to doing laundry?
- What measurement words relate to cooking?
- Where and what do we count during the day?

Mathematics:
Supporting English Learners As They Learn the Language of Math

Strands:	***Number Sense***	***Algebra and Functions***	***Measurement***	***Geometry***	***Mathematical Reasoning***

GETTING READY

Instructional Component(s): Information Delivery; In-Class Activity; Out-of-Class Activity

Strands: This activity can be used to develop familiarity with and deepen understanding of all mathematics strands or of individual strands.

Focus: Students consider how language development for a child who is an English learner would influence understanding and skills in mathematics.

AFTER PARTICIPATING IN THIS ACTIVITY

Students will demonstrate ***knowledge*** and ***skills*** that are consistent with an increased understanding and application of the concepts addressed in this activity.

Students will demonstrate an understanding of*:*

- How language development and learning English can influence how children demonstrate the math foundations (*Standards 1 & 5*)*
- The stages of learning a second language (*Standard 1*)*
- The foundations in mathematics, including strands, substrands, foundations, and examples of foundations (introductory understanding) (*Standards 1, 5, & 6*)*
- Strategies used to support children who are English learners in developing skills and concepts in math (*Standards 1, 4, & 5*)*

Students will be able to:

- Recognize the stages of learning a second language, related to developing skills and concepts in math (*Standard 1*)*
- Support children who are English learners develop skills and concepts in math (*Standards 1, 4, & 5*)*
- Identify math words in different languages that would be helpful to use and for children to see in the preschool environment (*Standards 4 & 5*)*
- Observe and document teaching strategies to support children at each stage of language development to support learning of math concepts (*Standards 1 & 5*)*

*See Appendix A

Mathematics:
Supporting English Learners As They Learn the Language of Math

Before you start

This activity asks students to develop ideas for supporting all children in the process of building knowledge and skills in mathematics as well as specifically focusing on children who are English learners. While this steps into the realm of curriculum development, it is included in this instructional guide because it addresses the crucial issue of math development for English learners.

It will be helpful to be familiar with the English-language development foundations as you guide your students through this activity.

Getting it started

Slide 2

This activity requires that students work with four stages that young children will demonstrate as they go through the process of bilingual language learning. These are available on pages 105 and 106 of the *California Preschool Learning Foundations, Volume 1*(PLF, V1). In addition, CDE has published *Preschool English Learners: A Resource Guide*. Chapter 5 of this publication presents more in-depth information regarding the stages that children pass through as they learn a second language.

It may be helpful for students to have a summary sheet of the foundations so that they can focus on the specific content you are targeting in this activity.

If you have the publication available, you can review the stages in Chapter 5 of CDE's *Preschool English Learners: A Resource Guide*. Be sure to emphasize the following points:

- A major practice emphasized throughout the resource guide is that it is critical to honor the home language as children learn English.
- Much of preschool math is related to learning words and associating those words with objects or processes. This means that any strategies to support children who are English learners as they develop language and literacy in English apply to developing skills and concepts in math as well.

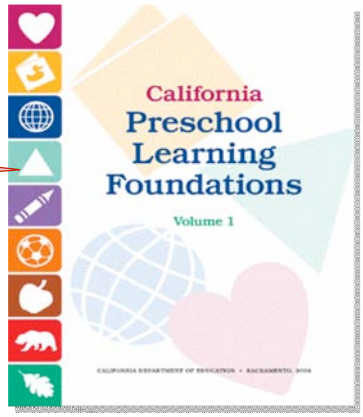
Keeping it going  Slides 3-4	Introduce students to the table provided as a handout following this activity. Ask students to think about the following question: • How can adults support children in each of these stages of learning English in ways that would *specifically help them learn the language of math* in English? In pairs or small groups, they can be asked to work through the entire table or they can be assigned to specific stages or strands.
Another way	If students need some support in initiating this activity, you might mention the following examples: • Posting written words in the environment in the home language of the children paired with English • Narrating what an adult or another child is doing • Using songs or finger-plays in the home language and English that relate to the math foundations
Putting it together	After they have gone through the table, ask them to share their ideas with the group.
Taking it further	Consider collecting and combining these ideas to give to students as a resource for their work.
Taking it out of class	Following class review of the initial information, ask students to work on this as an observation activity. Ask students to do an observation in a preschool classroom, paying attention to environment, interactions, and activities.

	What are some intentional teaching strategies that can be used for children at each of these levels to support learning of math concepts?				
	Number Sense	Algebra and Functions	Geometry	Measurement	Mathematical Reasoning
Use Of Home Language Children will use their home language when and where it works.					
Observational and Listening Period Children are quiet but listening and watching and possibly trying out gestures and sounds					
Telegraphic And Formulaic Speech Children might use important words, such as labels and actions, but omit many other words.					
Fluid language speech Children can use full sentences in a variety of contexts, but are still learning and expanding their knowledge of the language.					

Mathematics Domain

California Preschool Learning Foundations
Volume 1

Mathematics



Published by the
California Department of Education (2008)

Supporting English Learners as They Learn the Language of Math

- Honor the home language as children learn English.
- Math is about learning words and associating those words with objects or processes.
- Strategies used to support children who are English learners in developing language and literacy in English apply to developing skills and concepts in math.

Supporting English Learners as They Learn the Language of Math

What *intentional teaching strategies* can be used to support children in learning the language of math at each of these stages?

- Home Language Use
- Observational and Listening Period
- Telegraphic and Formulaic Speech
- Fluid Language use

Supporting English Learners as They Learn the Language of Math

What are some intentional teaching strategies that can be used for children at each of these stages to support learning the language of math?

	Number Sense	Algebra (classification and patterns)	Geometry	Measurement	Mathematical Reasoning
Home Language Use					
Observational and Listening Period					
Telegraphic and Formulaic Speech					
Fluid Language Use					

Mathematics:
Deepening Understanding: Linking Preschool Learning Foundations and Kindergarten Content Standards Related to Mathematics

Strands:	***Number Sense***	***Algebra and Functions***	***Measurement***	***Geometry***	***Mathematical Reasoning***

AFTER COMPLETING THIS ACTIVITY

Students will demonstrate ***knowledge*** and ***skills*** that are consistent with an increased understanding and application of the concepts addressed in this activity.

Students will demonstrate an understanding of:

- Expectations for kindergartners, as outlined in the California *Mathematics Content Standards for California Public Schools—Kindergarten*(introductory understanding) (*Standards 1, 5, & 6*)*
- Expectations for preschool children, as described in the mathematics domain of the PLF, V1, including strands, substrands, and foundations (*Standards 1, 5, & 6*)*
- Similarities and differences in expectations for children's developing knowledge and skills in math (*Standards 1 & 5*)*

Students will be able to:

- Critically review the foundations in mathematics to understand expectations for preschool children at or around 48 and 60 months of age or at the end of their first or second year of preschool and when in a high quality preschool program with adequate support for English learners or special needs (*Standards 1, 5, & 6*)*
- Critically review the California *Mathematics Content Standards for California Public Schools—Kindergarten* to understand expectations for preschool children (*Standards 1, 5, & 6*)*
- Identify and discuss similarities and differences in expectations for children's development and engagement with math (*Standard 5*)*

* See Appendix A

Mathematics:
Deepening Understanding: Linking Preschool Learning Foundations and Kindergarten Content Standards Related to Mathematics

Before you start

In this project, students become familiar with the California Department of Education's *Mathematics Content Standards for California Public Schools—Kindergarten.* You may want to remind students that they can download the standards on the California Department of Education's Web site: http://www.cde.ca.gov/be/st/ss/.

Deepening understanding

Slides 2-3

Review California Department of Education's *Mathematics Content Standards for California Public Schools—Kindergarten.*

Ask students to write a reflection paper relating to:

- What are the differences in expectations for preschool and kindergarten students relating to math skills and concepts?
- What is the rationale for these differences?
- What are some ways that their engagement with math would be the same and/ or different?

Mathematics Domain

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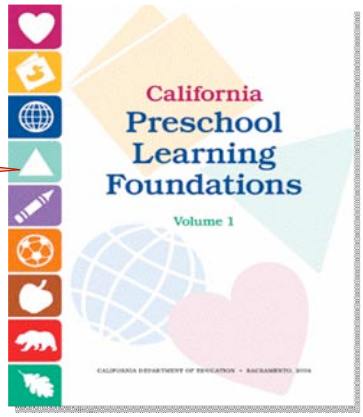

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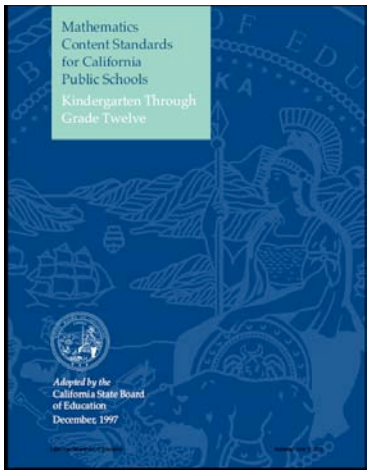
Linking Preschool Learning Foundations and Mathematics Content Standards



Linking Preschool Learning Foundations and Mathematics Content Standards

Write a reflection paper discussing:

- The differences in expectations for preschool and kindergarten students relating to mathematics skills and concepts.
- The rationale for these differences.
- Some ways that their engagement with mathematics would be the same and/or different.

Mathematics:
Deepening Understanding: SRCD Social Policy Report on Mathematics Education for Young Children

Strands:	***Number Sense***	***Algebra and Functions***	***Measurement***	***Geometry***	***Mathematical Reasoning***

AFTER COMPLETING THIS ACTIVITY

Students will demonstrate ***knowledge*** and ***skills*** that are consistent with an increased understanding and application of the concepts addressed in this activity.

Students will demonstrate an understanding of*:*

- Recent research and social policy, related to mathematics and young children (*Standards 1, 5, & 6*)*
- How to foster math learning through play (*Standards 4 & 5*)*

Students will be able to:

- Review a current Social Policy Report, focusing on mathematics *(Standard 6)**
- Identify and describe concerns about early childhood math education *(Standards 1, 5, & 6)**
- Identify ways to foster math learning through play *(Standards 1, 4, & 5)**

* See Appendix A

Mathematics:
Deepening Understanding: SRCD Social Policy Report on Mathematics Education for Young Children

Before you start

In late 2008, the Society for Research in Child Development (SRCD) published a Social Policy Report called *Mathematics Education for Young Children: What It Is and How to Promote It.*

This report is available online at www.SRCD.org. On the SRCD's home (welcome) page, click on publications. Next click on social policy report, and then click on current issues. The article is listed in 2008 publications and can be downloaded from the site.

Deepening understanding

Slides 2-3

Ask students to read this paper and respond to the following questions:

- What are four main concerns about early childhood math education?
- How does poverty affect children's math experiences?
- Identify five new things that surprised you or that you found particularly interesting.
- What are three things that teachers can do to become intentional teachers while fostering math learning through play?

Additional or different questions can be addressed, depending on the focus of a particular student, class, or program.

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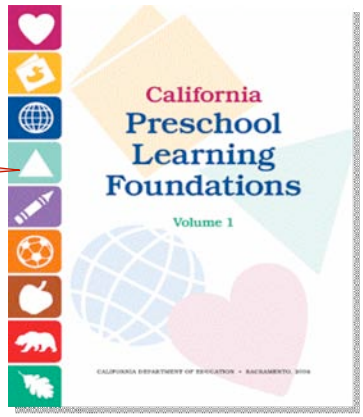

Published by the
California Department of Education (2008)

Mathematics Education for Young Children: What It is and How to Promote It

Herbert P. Ginsburg
Joon Sun Lee
Judi Stevenson Boyd

Abstract

Effective mathematics education for young children (approximately ages 3 to 5) seems to hold great promise for improving later achievement, particularly in low-SES students who are at risk of inferior education from preschool onwards. Yet there is limited understanding of what preschool and kindergarten mathematics education entails and what is required to implement it effectively. This paper attempts to provide insight into three topics central to understanding and improving early childhood mathematics education in the United States. First, we examine young children's mathematical abilities. Cognitive research shows that young children develop an extensive everyday mathematics and are capable of learning more and deeper mathematics than usually assumed. The second topic is the content and components of early childhood mathematics education. We show that the content of mathematics for young children is wide-ranging (number and operations, shape, space, measurement, and pattern)

SRCD Social Policy Report

- What are *four* main concerns about early childhood mathematics education?
- How does poverty affect children's math experiences?
- Identify *five* new things that surprised you or that you found particularly interesting.
- What are *three* things that teachers can do to become intentional teachers while fostering math learning through play?

Mathematics:
Deepening Understanding: Exploring Cultural Influences on Learning Mathematics

Strands:	***Number Sense***	***Algebra and Functions***	***Measurement***	***Geometry***	***Mathematical Reasoning***

AFTER COMPLETING THIS ACTIVITY

Students will demonstrate ***knowledge*** and ***skills*** that are consistent with an increased understanding and application of the concepts addressed in this activity.

Students will demonstrate an understanding of*:*

- The ways that context and culture, including experiences with our communities, language, and family, influence how children (and we as adults) learn math (*Standard 1*)*
- How children gain knowledge and understanding by building on prior experiences with their family and community (*Standard 1*)*
- Why familiar contexts and materials help children to apply mathematical concepts and use strategies (*Standards 1 & 5*)*

Students will be able to:

- Consider and identify ways in which culture and context influence development in mathematics (*Standards 1 & 5*)*
- Describe how cultural communities influence mathematics (*Standards 1, 2, & 5*)*

* See Appendix A

Mathematics:
Deepening Understanding: Exploring Cultural Influences on Learning Mathematics

Before you start

Many of the activities relating to the mathematics domain emphasize the strong relationship between culture and home language and learning the language of mathematics. In this project, students are asked to reflect on that issue, and to choose a research question from their reflections.

Deepening understanding

Slides 2-3

In *The Cultural Nature Of Human Development* (2003, Oxford Press, p. 266), Barbara Rogoff writes:

"Skilled use of cultural tools such as mathematics is intimately connected with many aspects of the practices and values of the communities in which they are used."

Ask students to think about some of the activities that have been completed in class and then to write a reflection on the quotation. Ask students to explore the following questions or other questions or issues that have come up in your class conversations in these reflections.

- What cultural practices and values have we explored in our conversations and activities relating to the math foundations?
- How does what children learn in early childhood settings about mathematics relate to what they experience at home?
- What are some important ways in which cultural communities influence mathematics?

When students have finished their reflection papers, you might decide to read and review them as a graded assignment or ask students to share their responses in class.

Depending on the previous exposure of your students to research design and development, you could then ask each student to develop a research question based on their reflections. The students could then conduct literature reviews for their research questions, again depending on their level of understanding of research design.

Mathematics Domain

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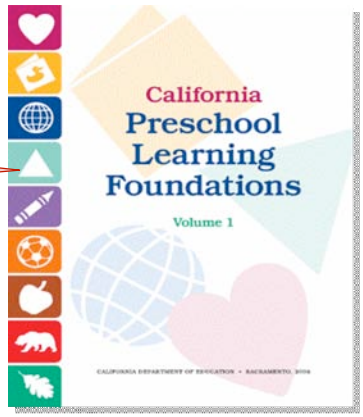

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California Department of Education (2008)

Exploring Cultural Influences on Learning Mathematics

"Skilled use of cultural tools such as mathematics is intimately connected with many aspects of the practices and values of the communities in which they are used."

- Rogoff (2003, p. 266)

Exploring Cultural Influences on Learning Mathematics

- What cultural practices and values have we explored in our conversations and activities relating to the mathematics foundations?
- How does what children learn in early childhood settings about mathematics relate to what they experience at home?
- What are some important ways in which cultural communities influence mathematics?

Mathematics:
Deepening Understanding: Perceptions of Success

Strands:	*Number Sense*	*Algebra and Functions*	*Measurement*	*Geometry*	*Mathematical Reasoning*

AFTER COMPLETING THIS ACTIVITY

Students will demonstrate ***knowledge*** and ***skills*** that are consistent with an increased understanding and application of the concepts addressed in this activity.

Students will demonstrate an understanding of*:*

- The research concerning children's success in math and their own (and others) attributions and beliefs about learning mathematics (*Standards 1 & 4*)*
- How their experiences learning mathematics may affect the way in which they help engage children with math (*Standards 1 & 6*)*

Students will be able to:

- Gather sources for and prepare a research paper about children's success in mathematics and attributions and beliefs about learning mathematics (*Standard 6*)*
- Discuss implications of research about children's success in math and related attributions and beliefs (*Standards 1 & 6*)*
- Reflect on their own experiences learning math (*Standards 4, 5, & 6*)*
- Consider how their own experiences affect how they support young children's development in mathematics (*Standards 4, 5, & 6*)*

*See Appendix A

Mathematics:
Deepening Understanding: Perceptions of Success

Before you start

There has been some interesting research done regarding the relationship between how children think about the source of success in math (talent versus effort) and their actual success. You might find some examples of this research to stimulate discussion prior to assigning this activity.

Deepening understanding

Slide 2

Ask students to write a review of research regarding the relationship between children's success in mathematics and their attribution of that success to effort and/or talent. This could be focused either on children's attributions or parents' attributions. Each student could find one study to describe or, depending on the level of prior exposure to research studies, develop a particular question on the topic for a more extensive literature review.

Ask students specifically to look for cultural and/or gender differences that influence that attribution.

Ask them to discuss implications for working with young children as they develop math skills and knowledge related to the foundations.

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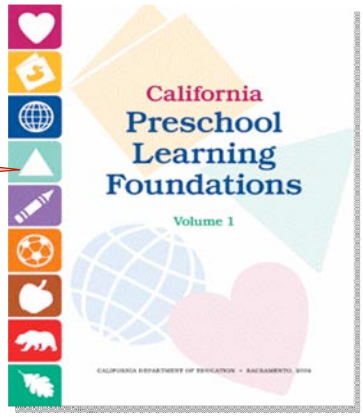


Published by the
California Department of Education (2008)

Perceptions of Success

- Write a research review describing the relationship between children's success in mathematics and their attributions of that success.
- Consider cultural and/or gender differences that may be influential.
- Discuss the implications for working with young children as they develop math skills and knowledge related to the foundations.

Appendix A

NAEYC Professional Preparation Standards

NBPTS (National Board of Professional Teaching Standards)

	Standard 1. Promoting Child Development and Learning	*Standard 2. Building Family and Community Relationships*	*Standard 3. Observing, Documenting, and Assessing to Support Young Children and Families*	*Standard 4. Using Developmentally Effective Approaches to Connect with Children and Families*	*Standard 5. Using Content Knowledge to Build Meaningful Curriculum*	*Standard 6. Becoming a Professional*
Standard 1. Understanding Young Children	X			X		
Standard 2. Equity, Fairness, and Diversity				X		
Standard 3. Assessment			X	X		
Standard 4. Promoting Child Development and Learning	X			X		
Standard 5. Knowledge of Integrated Curriculum	X		X	X	X	
Standard 6. Multiple Teaching Strategies for Meaningful Learning				X	X	
Standard 7 . Family and Community Partnerships		X				
Standard 8. Professional Partnerships						X
Standard 9. Reflective Practice						X

Appendix B

CORE NAEYC STANDARDS SUMMARY FOR EARLY CHILDHOOD PROFESSIONAL PREPARATION (NAEYC, July 2009)

Standard 1. Promoting Child Development and Learning

Students prepared in early childhood degree programs are grounded in a child development knowledge base. They use their understanding of young children's characteristics and needs and of the multiple interacting influences on children's development and learning to create environments that are healthy, respectful, supportive, and challenging for each child.

Key elements of Standard 1

1a: Knowing and understanding young children's characteristics and needs
1b: Knowing and understanding the multiple influences on development and learning
1c: Using developmental knowledge to create healthy, respectful, supportive, and challenging learning environments

Standard 2. Building Family and Community Relationships

Students prepared in early childhood degree programs understand that successful early childhood education depends upon partnerships with children's families and communities. They know about, understand, and value the importance and complex characteristics of children's families and communities. They use this understanding to create respectful, reciprocal relationships that support and empower families and to involve all families in their children's development and learning.

Key elements of Standard 2

2a: Knowing about and understanding diverse family and community characteristics
2b: Supporting and engaging families and communities through respectful, reciprocal relationships
2c: Involving families and communities in their children's development and learning

Standard 3. Observing, Documenting, and Assessing to Support Young Children and Families

Students prepared in early childhood degree programs understand that child observation, documentation, and other forms of assessment are central to the practice of all early childhood professionals. They know about and understand the goals, benefits, and uses of assessment. They know about and use systematic observations, documentation, and other effective assessment strategies in a responsible way, in partnership with families and other professionals, to positively influence the development of every child.

Key elements of Standard 3

3a: Understanding the goals, benefits, and uses of assessment
3b: Knowing about assessment partnerships with families and with professional colleagues
3c: Knowing about and using observation, documentation, and other appropriate assessment tools and approaches
3d: Understanding and practicing responsible assessment to promote positive outcomes for each child.

Standard 4. Using Developmentally Effective Approaches to Connect with Children and Families

Students prepared in early childhood degree programs understand that teaching and learning with young children is a complex enterprise, and its details vary depending on children's ages, characteristics, and the settings within which teaching and learning occur. They understand and use positive relationships and supportive interactions as the foundation for their work with young children and families. Students know, understand, and use a wide array of

developmentally appropriate approaches, instructional strategies, and tools to connect with children and families and positively influence each child's development and learning.

Key elements of Standard 4

4a: Understanding positive relationships and supportive interactions as the foundation of their work with children
4b: Knowing and understanding effective strategies and tools for early education
4c: Using a broad repertoire of developmentally appropriate teaching/learning approaches
4d: Reflecting on their own practice to promote positive outcomes for each child

Standard 5. Using Content Knowledge to Build Meaningful Curriculum

Students prepared in early childhood degree programs use their knowledge of academic disciplines to design, implement, and evaluate experiences that promote positive development and learning for each and every young child. Students understand the importance of developmental domains and academic (or content) disciplines in an early childhood curriculum. They know the essential concepts, inquiry tools, and structure of content areas, including academic subjects, and can identify resources to deepen their understanding. Students use their own knowledge and other resources to design, implement, and evaluate meaningful, challenging curricula that promote comprehensive developmental and learning outcomes for every young child.

Key elements of Standard 5

5a: Understanding content knowledge and resources in academic disciplines
5b: Knowing and using the central concepts, inquiry tools, and structures of content areas or academic disciplines
5c: Using their own knowledge, appropriate early learning standards, and other resources to design, implement, and evaluate meaningful, challenging curricula for each child.

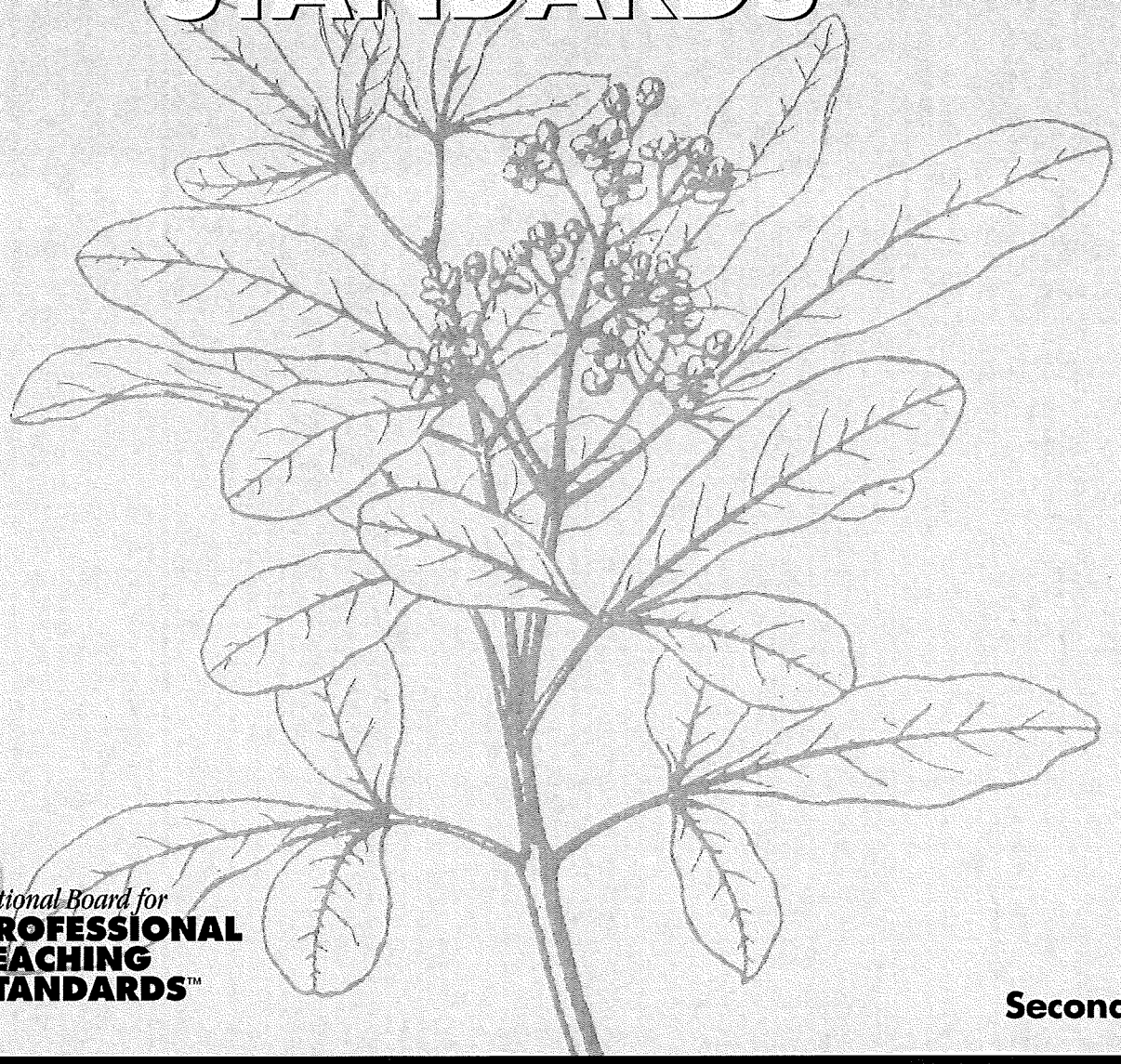
Standard 6. Becoming a Professional

Students prepared in early childhood degree programs identify and conduct themselves as members of the early childhood profession. They know and use ethical guidelines and other professional standards related to early childhood practice. They are continuous, collaborative learners who demonstrate knowledgeable, reflective, and critical perspectives on their work, making informed decisions that integrate knowledge from a variety of sources. They are informed advocates for sound educational practices and policies.

Key elements of Standard 6

6a: Identifying and involving oneself with the early childhood field
6b: Knowing about and upholding ethical standards and other professional guidelines
6c: Engaging in continuous, collaborative learning to inform practice
6d: Integrating knowledgeable, reflective, and critical perspectives on early education
6e: Engaging in informed advocacy for children and the profession

Early Childhood Generalist STANDARDS



National Board for
PROFESSIONAL
TEACHING
STANDARDS™

Second Edition

for teachers of students ages 3–8

Early Childhood/Generalist STANDARDS

(for teachers of students ages 3–8)

Second Edition

OVERVIEW

The National Board for Professional Teaching Standards has organized the standards for accomplished Early Childhood/Generalist teachers into the following nine standards. The standards have been ordered to facilitate understanding, not to assign priorities. They each describe an important facet of accomplished teaching; they often occur concurrently because of the seamless quality of accomplished practice. These standards serve as the basis for National Board Certification in this field.

I. **Understanding Young Children (p. 7)**
Accomplished early childhood teachers use their knowledge of child development and their relationships with children and families to understand children as individuals and to plan in response to their unique needs and potentials.

II. **Equity, Fairness, and Diversity (p. 11)**
Accomplished early childhood teachers model and teach behaviors appropriate in a diverse society by creating a safe, secure learning environment for all children; by showing appreciation of and respect for the individual differences and unique needs of each member of the learning community; and by empowering children to treat others with, and to expect from others, equity, fairness, and dignity.

III. **Assessment (p. 15)**
Accomplished early childhood teachers recognize the strengths and weaknesses of multiple assessment methodologies and know how to use them effectively. Employing a variety of methods, they systematically observe, monitor, and document children's activities and behavior, analyzing, communicating, and using the information they glean to improve their work with children, parents, and others.

IV. **Promoting Child Development and Learning (p. 19)**
Accomplished early childhood teachers promote children's cognitive, social, emotional, physical, and linguistic development by organizing and orchestrating the environment in ways that best facilitate the development and learning of young children.

V. **Knowledge of Integrated Curriculum (p. 29)**
On the basis of their knowledge of how young children learn, of academic subjects, and of assessment, accomplished early childhood teachers design and implement developmentally appropriate learning experiences that integrate within and across the disciplines.

VI. **Multiple Teaching Strategies for Meaningful Learning (p. 45)**
Accomplished early childhood teachers use a variety of practices and resources to promote individual development, meaningful learning, and social cooperation.

VII. **Family and Community Partnerships (p. 51)**
Accomplished early childhood teachers work with and through families and communities to support children's learning and development.

VIII. Professional Partnerships (p. 55)
Accomplished early childhood teachers work as leaders and collaborators in the professional community to improve programs and practices for young children and their families.

IX. Reflective Practice (p. 59)
Accomplished early childhood teachers regularly analyze, evaluate, and synthesize to strengthen the quality and effectiveness of their work.

The pages that follow provide elaborations of each standard that discuss the knowledge, skills, dispositions, and habits of mind that describe accomplished teaching in the field.